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Solid Fluorescent Materials

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A FLUORESCENT material, characteristically, is one which can absorb energy in some invisible form and emit a part of that energy as visible light. Such energy-converters are useful because the invisible forms of energy are sometimes easier to produce or control than is light to which human sense is alert, or they have other convenient properties that visible light lacks. For example, the direction and the intensity of a beam of electrons can be changed rapidly and easily; if a beam of varying intensity is swept rhythmically across a layer of fluorescent material that changes part of the energy of the electrons into light, there results a visible pattern, which may be a television image. An electric discharge in mercury vapor at low pressure is inefficient as a light source, because a large fraction of the electric power is inevitably used up in producing the resonance radiation of mercury, which lies in the ultraviolet, is useless as a direct illuminant and is absorbed by the glass wall enclosing the discharge. But this ultraviolet radiation can be salvaged by a thin layer of fluorescent material on the inside of the tube; the combination of a mercury discharge to produce ultraviolet and a fluorescent material to convert this radiation into visible light makes an efficient lamp, and one which has some flexibility as to color. By the use of fluorescent materials, the penetrating power of x-rays is made available for the direct internal examination of opaque objects. A mixture of a fluorescent material with a radioactive element which continually emits exciting particles has long been in common use as a self-luminous coating for the figures and hands of watches.

All these applications involve solid inorganic fluorescent materials, or *phosphors*. This review will be concerned only with these inorganic crystalline materials. The fluorescence of liquids and of biologic compounds is of no less interest,[1] and the fluorescence of gases is simpler to interpret,[2] but the phosphors as a group are at present perhaps more widely used and are being more actively studied than are the noncrystalline substances.

It is recorded that Japanese painters of the Sung Dynasty, nearly a thousand years ago, knew of a phosphorescent material made by firing oyster shells, and mixed this phosphor with their paints to produce magic pictures. In the western hemisphere, Vincenzo Cascariola of Bologna, a cobbler by trade and an amateur alchemist, is generally credited with the discovery of the first phosphor, by accident, in 1602.

[1] These topics are treated concisely by E. Hirschlaff, *Fluorescence and phosphorescence* (New York, 1938). A useful system of chemical analysis is based on fluorescence phenomena; see J. A. Radley and J. Grant, *Fluorescence analysis in ultraviolet light* (New York, 1933), and M. Haitinger, *Die Fluorescenz Analyse in der Mikrochemie* (Leipzig, 1937).

[2] See R. W. Wood, *Encyclopaedia Britannica* (ed. 14), Vol. 9, p. 422 ff.

His "Bologna stone" was, presumably, a fired impure barium sulfide, and his discovery was that this stone would glow in the dark after exposure to sunlight—that is, it showed phosphorescence. Some fifty years had elapsed, and the list of substances known to have this property had grown large, before it was recognized that phosphorescent materials are also generally fluorescent, yielding light during excitation as well as afterwards. With the invention of the spectroscope it became possible to analyze the exciting radiation and the emitted light and to look for relations between the two. This problem had the attention of Stokes and of Becquerel, in the middle of the last century. The early workers with radioactive substances, with x-rays and with electrons found a large group of responsive phosphors ready at hand. In the period of the discovery of these new agents of excitation, important work on the preparation and properties of phosphors was being done, notably by Lenard and his school at Heidelberg and by Nichols and others at Cornell University.[3]

The increasing interest in phosphors during the past few years can be attributed to two causes. Foremost is the fact that two new applications, television and fluorescent lighting, have come forward to demand fluorescent materials which are highly efficient and which have in addition certain other specified qualities. The search for phosphors suited to these uses has not been unsuccessful,[4] but it is only fair to say that, up to the present, progress has been made chiefly by improving the materials which were known many years ago. In the second place, fluorescent materials are currently of interest because the theory of the solid state of matter has reached a stage where it should be able to account for their properties. The earlier concepts of Lenard and others are being reworded in the language of the modern theory, some new fluorescence phenomena are being discovered and others rediscovered, and a diligent effort is being made, with some success, to link the diverse observations together on the basis of a general picture of how crystalline solids are constructed.[5] In a later section of this article one aspect of this effort will be considered briefly.

Almost all solids will fluoresce to some extent if the temperature and the exciting stimulus are properly chosen. If it be required that the energy conversion shall occur at room temperature with an efficiency of at least a few percent, still many different materials are available. Three classes of phosphors—the sulfides, the silicates and the tungstates—are most widely employed technically and have been most thoroughly studied. We shall use these classes to illustrate how the various phosphors are alike and how they differ from one another. Table I gives, by way of preview, a rough summary of the distinctive properties of these three types.

Preparation and Activation of Phosphors

Phosphors are usually made by firing, at some temperature a little short of the melting point, a mixture of basic ingredients usually with a small proportion of another element known as the "activator." Each matrix material requires

Table I. *Survey of the properties of three classes of phosphors.*

Phosphor Type	Sulfide	Silicate	Tungstate
Examples	$ZnS\cdot Cu$ $ZnS\cdot Ag$ $ZnS\cdot CdS\cdot Ag$ $CaS\cdot Bi$	$Zn_2SiO_4\cdot Mn$ $Cd_2SiO_4\cdot Mn$ $Mg_2SiO_4\cdot Mn$ $ZnBeSiO_4\cdot Mn$	$CaWO_4$ $CdWO_4$ $MgWO_4$ (no activator)
Approximate longest wave-length of exciting radiation	4100A	2800A	3100A
Color of fluorescence	Variable from blue through red	Variable from blue-green through red	Blue or blue-white
Approximately peak efficiency of energy conversion. Excitation by: ultraviolet electron beam α-particles	 50% 10% 25%	 50% 10%	 50% 10%
Type of decay	Complicated	Initially exponential, later complicated	Brief

[3] A wealth of data on the various phosphors is collected by P. Lenard, F. Schmid and R. Tomaschek, *Handbuch der Experimentalphysik* XXIII/1 and 2 (Leipzig, 1928) and by E. L. Nichols and E. Merritt, *Studies in luminescence* (Washington, 1912). The field is reviewed by H. Rupp, *Die Leuchtmassen und ihre Verwendung* (Berlin, 1937), by P. Pringsheim, *Fluorescenz und Phosphorescenz* (ed. 3 Berlin, 1928), by L. Vanino, *Die Leuchtfarben* (Stuttgart, 1935), by M. Curie, *Luminescence des corps solides* (Paris, 1934), and by M. Hirschlaff, ref. 1.

[4] H. W. Leverenz and F. Seitz, J. App. Phys. **10**, 479 (1939); R. N. Thayer and B. T. Barnes, J. Opt. Soc. Am. **29**, 131 (1939).

[5] Papers presented at the symposium held by the Faraday Society in September, 1938, published in its *Transactions* and also separately under the title *Luminescence*, are representative of this most recent phase.

its appropriate activator, though often different activators can be added to the same matrix to produce phosphors of different properties. Precipitated ZnS, for instance, is fired with a small trace of CuS (and perhaps with a volatile flux such as NaCl, to hasten the homogenization); the resulting colorless powder, denoted briefly by the symbol ZnS·Cu, is a phosphor which fluoresces green. Ag and Mn are also effective activators for ZnS. CaS is usually activated by Bi. Zinc silicate phosphor ("synthetic willemite"), made by firing ZnO and SiO_2 together, is activated specifically by Mn.

The efficiency of a phosphor increases with increasing activator content up through a maximum. For the sulfides, this maximum comes at a concentration of about 1 activator atom in every 10^4 atoms of the base material. For the silicates it is higher: about 1 atom out of every 200 should be Mn. It is plain from these figures that, particularly with the sulfides, extreme care is necessary if the final product is to contain only the desired activator and no adventitious impurities. Some of the possible impurities, notably Fe and Ni, are not merely inert but act as poisons,[6] inhibiting the fluorescence even at concentrations as low as 10^{-6}. The great importance of purity in the basic ingredients has been fully recognized only since the work of Lenard. The complicated and erratic behavior which some earlier investigators found with phosphors supposedly simple is probably to be blamed on the unsuspected presence of other activators or poisons. Phosphors are now commonly manufactured in rooms specially freed of dust, and "fluorescence pure" is acknowledged as a more critical specification than "spectroscopically pure."

The tungstates apparently are an exception to the rule that an activator is required. After suitable firing they are good phosphors without the addition of any intentional activator,[7] and their efficiency is not impaired as they are made more and more free of accidental impurities. But the firing is necessary, and it is not implausible that some of the components of the tungstate itself are displaced to anomalous positions in the lattice and are serving as activators. A similar problem is presented by ZnS. Very pure ZnS after firing exhibits a blue fluorescence,[8] which is attributed by some investigators to the presence of Zn atoms in excess of the formula ratio, and by others to the coexistence of two allotropic forms of the sulfide. As Cu is added this blue emission decreases in prominence and the green fluorescence characteristic of ZnS·Cu increases to its maximum. The tungstates require no foreign activator, but it cannot be guaranteed that in the fired product the strict combining ratios are preserved and every atom is at its appointed lattice site in a single homogeneous phase.

Emission Spectra

The light emitted by a phosphor usually covers continuously a broad band in the visible spectrum. Its subjective color depends, of course, on the breadth of this band and on its detailed structure. A few examples will show the range. $CaWO_4$ fluoresces pale blue, as does $CdWO_4$. $MgWO_4$ yields a cold white light. Zn_2SiO_4·Mn (synthetic willemite) in its usual crystalline form fluoresces green. If this same material is made into a glass by melting and quick cooling it then fluoresces, not green, but a brilliant yellow; when the ratio of SiO_2 to ZnO in the mix is increased, melting and quenching produces another phosphor which emits in the red.[9] If, in crystalline Zn_2SiO_4·Mn, some of the Zn is replaced by Be, the color is shifted from the green toward the red, and farther to the red, the higher the Be content. Among the sulfides, we have remarked that unactivated ZnS yields blue light, while ZnS·Cu fluoresces green. ZnS·Ag fluoresces blue, as does CaS·Bi. The mixed sulfide of Zn and Cd, activated with Cu or Ag, has an emission peak between the blue and the red, which lies farther toward the red as the Cd proportion is increased.[10] Spectral dis-

[6] See, for example, J. W. Marden and G. Meister, Trans. Ill. Eng. Soc. **34**, 503 (1939).

[7] A trace of Pb, however, seems to have an activating effect; see F. E. Swindells, J. Opt. Soc. Am. **23**, 129 (1933).

[8] Discovered by E. Schleede, Zeits. f. anorg. allgem. Chemie **48**, 277 (1935).

[9] Experiments of G. R. Fonda in this laboratory (awaiting publication).

[10] Discovered by A. Guntz, Comptes rendus **77**, 479 (1923). The curves of Fig. 2 are taken from K. Kamm, Zeits. f. Physik **30**, 333 (1937).

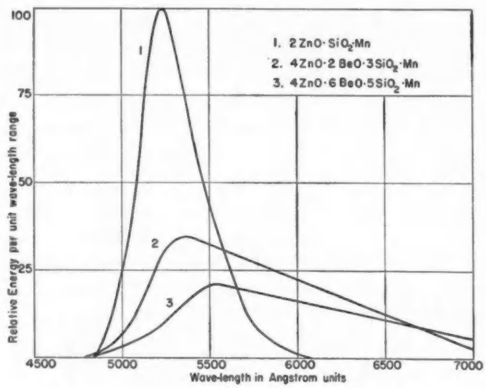

FIG. 1. Spectral distribution of the fluorescence of zinc silicate and zinc-beryllium silicate phosphors, showing the continuous shift toward the red as the Be content is increased. [From Leverenz and Seitz. ref. 4.]

tribution curves for several of these cases appear in Figs. 1, 2 and 3.

These detailed examples are intended to illustrate, first, that almost any hue can be achieved, and second, that the spectral distribution of the emitted light varies with the activator, with the base material and with the physical state of the phosphor.

For various applications various colors are required. Other things being equal, the most efficient phosphor from a visual standpoint is one that emits in the yellow-green region, around 5600A, because the eye is most sensitive to light of this wave-length—here 1 w of light energy corresponds to the maximum number of lumens, as is shown by Fig. 4. Fluoroscopic screens for x-ray work, where the available intensity is low, are therefore preferably green-fluorescing. ZnS·Cu screens have, in fact, come to replace the older screens of $CaWO_4$ and of barium platinocyanide (which fluoresce blue) since Levy and West found[11] that a minute trace of Ni would inhibit the phosphorescence without noticeably decreasing the efficiency of fluorescence. X-ray intensifying screens, on the other hand, may advantageously yield blue light to which the photographic film is more sensitive. For television picture tubes, black and white is commonly held to be the most satisfactory combination. To get the desired tone, along with other necessary qualities such as the right degree of phosphorescence, stability during life and the like, mixtures of several phosphors are employed. In fluorescence lighting for general illumination, as contrasted with decorative lighting, a spectral distribution resembling that of daylight or of an incandescent lamp is necessary. In the commercial fluorescent lamps,[12] whites are achieved by mixing phosphors; the different colors of the individual grains can be seen with a magnifying glass (see also Fig. 3).

AGENTS OF EXCITATION

It is an interesting and important question, what radiation or other energy is effective in exciting phosphors. One rule can be stated with considerable assurance—since Stokes first enunciated it in 1852 no significant exceptions have been found: the wave-length of the exciting radiation must be less than the wave-length of the emitted light. The emitted quanta, in other

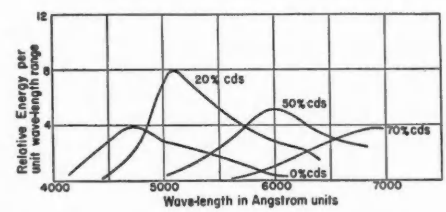

FIG. 2. Change in spectral distribution with change of Cd content in the mixed sulfide phosphor ZnS·Cds·Ag. [From Kamm, ref. 10.]

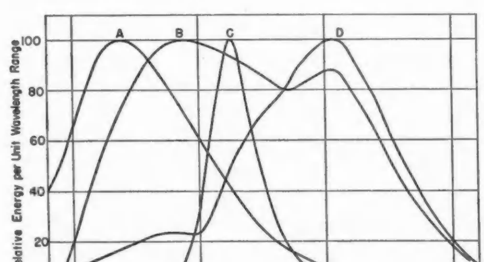

FIG. 3. Spectral distribution of fluorescent light from four commercial lamps: *A*, "blue"; *B*, "daylight"; *C*, "green"; *D*, "white." [From Inman, ref. 12.] The line spectrum of the mercury discharge is not shown.

[11] L. Levy and D. W. West, Brit. J. Radiology **8**, 184 (1935).

[12] See G. E. Inman, Trans. Ill. Eng. Soc. **34**, 65 (1939), from which the curves of Fig. 3 are adapted.

words, have less energy than the absorbed quanta. Since the emitted light lies in the visible region, radiation in the red end of the spectrum will not excite fluorescence. Exposure to a ruby photographic safelamp, for instance, does not cause fluorescence in either the sulfides, the silicates or the tungstates. The radiation from an incandescent lamp, extending across the visible region down to about 3200A, excites the sulfides (the exciting radiation is chiefly in the near ultraviolet, around 3600A) but not the silicates or the tungstates. The resonance radiation from a low pressure mercury discharge in quartz or ultraviolet-transmitting glass, well down in the ultraviolet at 2537A, excites all three. Fig. 5 shows some typical excitation

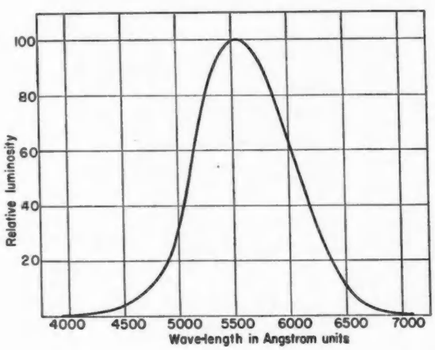

FIG. 4. Relative response of the human eye to light of various wave-lengths.

curves, the efficiency of production of fluorescence light being plotted as a function of the wave-length of the exciting radiation. It is at present a subject for speculation, why the sulfides, for example, should be responsive to quanta of longer wave-length than are the silicates.

There is a striking demonstration to show that the excitation, when the source is a low pressure mercury discharge in a tube transparent to ultraviolet, is predominantly by the resonance radiation. This radiation, emitted when a mercury atom reverts from an excited state to its normal state, is strongly absorbed by other mercury atoms in their normal unexcited state. A little mercury vapor placed between the lamp and the fluorescent screen (green-fluorescing $Zn_2SiO_4 \cdot Mn$, preferably, since this is practically inert to the longer wave-length nonresonance radiation) will intercept this energy and cast a definite shadow. The cloud of vapor rising from an open dish of mercury at room temperature is readily made evident by this means[13] (Fig. 6).

In the direction of shorter wave-lengths, toward quanta or other entities of larger energy, many of the phosphors remain phosphors. That is, many of them which fluoresce brightly under ultraviolet radiation also fluoresce to a noticeable extent under ultraviolet radiation of shorter wave-length, and are also excited by x-rays, by high speed electrons and by high speed ions, such as α-particles or the projectiles emergent from a cyclotron. The efficiency of energy conversion, however, usually varies widely with the wave-length of the exciting radiation or the nature of the exciting particles, in a manner at present unpredictable. The phosphor suitable for each energy source must therefore be determined by experiment; some examples appear in the following section. The spectral distribution of the emitted light is largely independent of the type of exciting agent.

EFFICIENCY OF ENERGY CONVERSION

As Fig. 5 shows, the efficiency of a phosphor is a function of the wave-length of the exciting radiation, and for different phosphors the peak

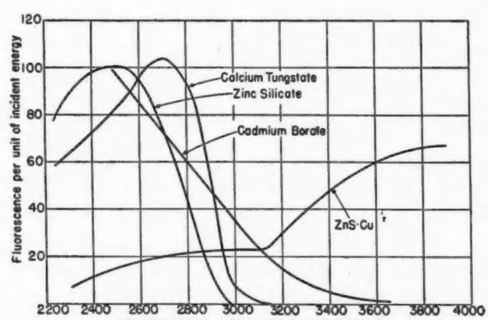

FIG. 5. Excitation curves for several typical phosphors. The curve for ZnS·Cu is taken from unpublished work of G. R. Fonda and F. B. Quinlan in this laboratory; the others are from the paper of Thayer and Barnes (ref. 4). The ordinates are not comparable from one curve to another.

[13] This experiment is the basis of a sensitive method of testing for stray mercury vapor in the atmosphere, first developed by C. W. Hewlett of this laboratory. See T. T. Woodson, Rev. Sci. Inst. **10**, 308 (1939).

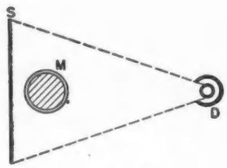

FIG. 6. Resonance radiation from the shielded mercury discharge at D is intercepted by mercury vapor rising from the open dish M, and the shadow of this vapor is cast on the zinc silicate screen S.

or peaks lie at different wave-lengths. The sulfides as a group are excited efficiently by radiation in the neighborhood of 3600A, whereas at 2537A their efficiency has fallen well below that of the silicates and the tungstates. The sulfides are therefore not particularly suitable for use in fluorescent lamps where a low pressure mercury discharge is the source of exciting radiation, but they have found some use in conjunction with mercury arcs at higher pressures, where the line at 3650A is generated in large quantity. Magnesium silicate, on the other hand, is not responsive to wave-length 2537A, but is efficiently excited by the resonance radiation of the rare gases, at wave-lengths shorter than 1500A. As another example, certain alkali halides, when suitably activated, respond efficiently to x-ray excitation but are only weakly fluorescent under exposure to ultraviolet.

The best sample of any of the three types of phosphors listed in Table I, when excited by ultraviolet radiation of the wave-length to which it is most responsive, gives out nearly one quantum of fluorescent light for each quantum of ultraviolet it absorbs. On the basis of quanta, the peak efficiency is in the neighborhood of 100 percent, or approximately 50 percent on the usual basis of energy conversion. Table II gives the measured quantum efficiencies of several typical phosphors when excited by mercury resonance radiation. Processes in which a single ultraviolet quantum excites more than one fluorescence quantum are not unthinkable, but there is no evidence that they occur in phosphors. With x-ray excitation, a rough guess gives, for the best fluoroscopic screens, an energy-conversion efficiency of the order of a few percent—a thin layer of phosphor absorbs, of course, only a little of the energy of incident hard x-rays. When electrons are used, the efficiency, curiously, goes up regularly as the energy of the electrons is increased.[14] Efficiencies as high as 10 percent have been reported. Taking this figure, a single 10,000-v electron produces some 400 quanta of visible light. The energy of α-particles is converted into light, by the best sulfide phosphors, with an efficiency as high as 25 percent[15]—a single 8 million-volt α-particle yields about half a million quanta. These efficiencies are remarkably high, as compared with the over-all efficiency of the tungsten filament lamp (3 to 5 percent), the sodium lamp (8 to 12 percent) or the high pressure mercury lamp (6 to 8 percent).

TABLE II. *Efficiencies of phosphors used in fluorescent lamps, excited by the resonance radiation of mercury.* [*Adapted from Thayer and Barnes, ref. 4.*]

PHOSPHOR	QUANTA OF FLUORESCENCE LIGHT PER INCIDENT QUANTUM OF 2537A	WATTS OF FLUORESCENCE LIGHT PER INCIDENT WATT OF 2537A
Calcium tungstate	0.70	0.41
Magnesium tungstate	.70	.37
Zinc silicate	.74	.35
ZnBe silicate	.53	.23
Cadmium silicate	.55	.23
Cadmium borate	.66	.27

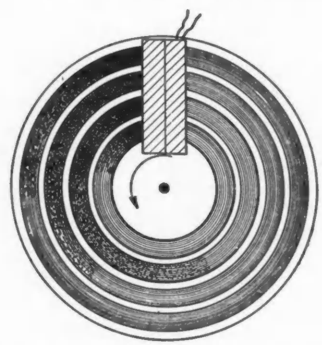

FIG. 7. A simple phosphoroscope for demonstrating the different lengths of afterglow periods of different phosphors. The materials are coated as concentric rings on a disk which can be turned past a narrow source of exciting radiation, here shown as a mercury discharge in a shielding can.

PHOSPHORESCENCE

No sharp distinction is possible between fluorescence, during excitation, and phosphores-

[14] For examples, see W. B. Nottingham, J. App. Phys. **10**, 73 (1939).

[15] Much higher efficiencies have in fact been reported, but this figure, taken from J. Chariton and C. A. Lea, Proc. Roy. Soc. **A122**, 304 (1929), seems the most reliable.

cence, after excitation. At least for simple phosphors of the three classes considered here, the spectral distribution of the emitted light is the same in both phases of luminescence, and the brightness decays, in the phosphorescence period, continuously downward from its value during excitation.

Among the materials of Table I, phosphorescence of many degrees of persistence is to be found. A sample which, when stationary, seems to go dark immediately after the excitation is stopped, may reveal a phosphorescence lasting

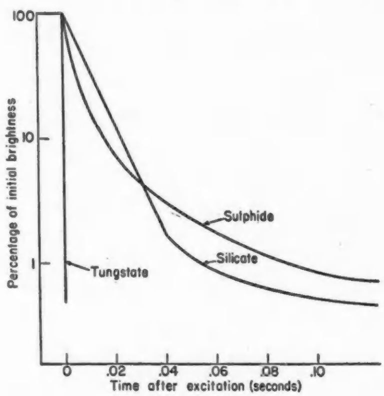

FIG. 8. Schematic decay curves for the three types of phosphors listed in Table I.

a large fraction of a second if it is moved rapidly past the exciting source into a comparatively dark place. For demonstration purposes, a simple phosphoroscope is made by painting concentric rings of different phosphors on a disk which can be turned at various speeds (the step-up gear box of a small hand-operated grinding wheel is a convenient drive) past a narrow source of exciting radiation. The length of the visible arc is an indication of the duration of the afterglow (Fig. 7). This general method of observation suffices for detecting phosphorescence down to periods of the order of 10^{-4} sec. For studying phosphors that decay more rapidly, an electro-optical shutter involving a Kerr cell has been used.

Typical decay curves, at room temperature, for the three types of materials of Table I are shown schematically in Fig. 8. For most practical purposes a phosphor that decays to insensible brightness in less than 10^{-3} sec might as well not be phosphorescent at all. The tungstates are in this class; they have so short an afterglow that no one has bothered to determine the law relating their brightness to the time after excitation.

The silicates at first decay exponentially, following the same law as governs the decay of a simple radioactive element or the discharge of a condenser through a resistance. Later the decay is slower and follows a different law. It is this lasting component that one sees after stopping the excitation of a stationary silicate sample; the exponential phase is over in less than a second. The prominence and the course of the lasting component vary markedly from one sample of silicate to another, depending on the time and temperature of firing, the rapidity of cooling, the grain size, the intensity and duration of excitation, and other parameters. In contrast, the initial exponential decay is little sensitive to these changes. Its time constant is different for the different silicate types ($\sim 14 \times 10^{-3}$ sec for $Zn_2SiO_4 \cdot Mn$, $\sim 30 \times 10^{-3}$ sec for $Cd_2SiO_4 \cdot Mn$, as examples), but for different silicates of the same type it is invariant.

The sulfides as a group do not show any initial exponential decay. All their phosphorescence is apparently of the same sort as the lasting phosphorescence of the silicates. The initial decay rate is higher, the more intense the excitation. The shape of the entire decay curve varies markedly among different samples of the same type (for example, $ZnS \cdot Cu$), and it is only an exceptional decay curve that can be represented by a mathematical law indicating a simple mechanism.

All these remarks apply to phosphors at room temperature that have been excited for at least

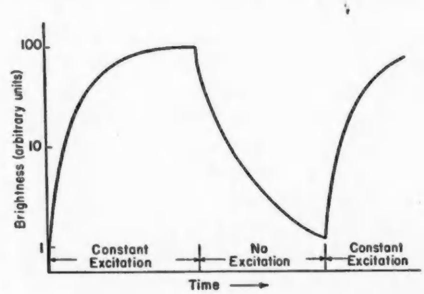

FIG. 9. Schematic build-up and decay curve for a sulfide phosphor subjected to cyclic excitation.

a few seconds. If the excitation is very brief, a phosphor which shows persistent decay does not have time to come to equilibrium with the source. There is a build-up period (Fig. 9) which can be related at least qualitatively with the period of decay (the condenser-resistor analogy is suggestive, but not quite apt). As the intensity and duration of the excitation are increased, the total light stored up in a phosphor and recoverable during phosphorescence also increases, not without limit, but approaching a saturation value. This "maximum light-sum," as Lenard calls it, is variable from one sample to another, as is the course of the persistent phosphorescence.

In practical applications all degrees of persistence are required. In television picture tubes, each point on the screen must decay to negligible brightness in a time shorter than the time between transits of the scanning electron beam; otherwise, bright moving objects would appear to have comet-like tails behind them. Similarly, the x-ray diagnostician requires a fluoroscopic screen with a short memory. On the other hand, in fluorescence lighting with alternating current, a phosphor with pronounced afterglow acts as a "flywheel" to keep the light level up during the parts of the cycle when the electric discharge is out, and thus reduces undesirable flicker. It is conceivable, finally, that a material with extremely long phosphorescence might be used for coating highways, to absorb light without cost from the sky during the day and dispense light without cost to the traveling public during the night.

Effects at High and Low Temperatures

When phosphors are studied at temperatures above and below room temperature, the phenomena become so varied that it is possible to summarize here only the most general facts. First, as the temperature of a phosphor is raised, its efficiency decreases, and at some temperature usually short of incandescence it ceases to fluoresce. The temperature range over which this decline occurs varies from one phosphor to another. Fig. 10 shows that for $Zn_2SiO_4 \cdot Mn$ the efficiency falls off at a lower temperature, the higher the Mn content.[16] The rate of decay of the sulfides becomes larger at higher temperatures, as does the rate of decay of the lasting phosphorescence of the silicates. The rate of the initial exponential decay of the silicates, contrastingly, remains nearly independent of temperature. These effects of increased tem-

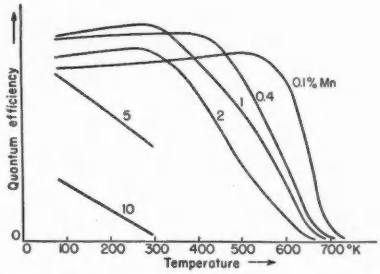

FIG. 10. Showing that the efficiency of $Zn_2SiO_4 \cdot Mn$ phosphors is maintained to higher temperatures, the lower the percentage of activator (ref. 16).

perature on efficiency and on phosphorescence are easily demonstrated with samples coated on an electric hot-plate or on a sheet of metal which can be heated with a dark Bunsen flame.

In the direction of lower temperatures the efficiency of ordinary phosphors does not markedly change. But many other materials which show only slight fluorescence at room temperature become fairly efficient at the temperature of liquid air; cloth, paper and blackboard chalk may be cited as examples.

The phosphorescent behavior of ordinary phosphors at low temperatures is striking but not surprising. Raising the temperature increases the rate at which stored-up light leaves the phosphor; conversely, lowering the temperature decreases the rate. If a sulfide sample, while it is still glowing strongly after excitation, is put into liquid air, it goes dark immediately and stays dark until the temperature is again raised to the neighborhood of room temperature. It is possible in this way to keep the light captive for weeks—this is the method by which Florida sunshine was transported to the 1939 New York World's Fair.[17] The lasting component of the decay of silicates can also be held in check by lowering the temperature, though the initial exponential decay goes on at the same rate as at room temperature.

Theory of Sulfide Phosphors

It is now in place to point out some of the requirements that any acceptable theory of crystal phosphors must meet, and to indicate the direction in which one at present looks for an explana-

[16] Fig. 10 is taken from unpublished work of G. R. Fonda and C. Zener in this laboratory.

[17] New York Times, Aug. 30, 1939, p. 12, col. 2.

tion of fluorescence phenomena. Specifically, the sulfides will be considered.

Each type of sulfide and, in fact, each sample of a given type has its own peculiarities of behavior. Any model must have enough flexibility to comprise these individual diversities. Experience with the production of light from gases accustoms us to think naturally of electrons which absorb energy and are thereby raised to excited states, a quantum of light being emitted when each electron returns to its normal level of energy. This idea is inevitably to be taken over bodily into the theory of sulfide phosphors. The activator may be expected to play an important part in these transitions of electrons.

It would be convenient if we could say that the activator atoms absorb the incident energy directly and re-emit a part of it as light, the surrounding matrix material serving only to perturb the energy levels of electrons in the activator atoms so that a continuous spectrum, rather than a line spectrum, is emitted. Two considerations restrain us from accepting this pleasantly uncomplicated hypothesis. One is the fact that the sulfide phosphors are highly efficient. It might be supposed, when the excitation is by ultraviolet radiation, that the activator atoms are peculiarly apt to absorb this radiation, as compared with atoms of the base material. But this is not at all a plausible assumption for the case of excitation by electrons, and it is even less plausible when massive atomic nuclei are used as exciters. An α-particle ploughing through a ZnS·Cu crystal will encounter 10^4 Zn or S atoms for each Cu atom it meets, and each atom regardless of kind will take approximately the same quantity of energy from the particle. Yet, as much as 25 percent of the total energy can be recovered as visible light, and a Cu atom is somehow involved in the production of every quantum of this light. Clearly it must be true that energy absorbed in any part of the lattice can be transferred to the vicinity of a Cu atom. The other consideration concerns the phenomenon of phosphorescence, the fact that the light can remain latent for a time which is longer, the lower the temperature. Not only must the energy of excitation be able to move around in the crystal, but places must exist where it can be trapped temporarily, until it is freed by thermal agitation.

The modern picture of the behavior of electrons in solids[18] supplies just the concepts needed for these items. If one imagines a pure ZnS crystal made by bringing the constituent atoms closer and closer together, he finds that the energy levels for valence electrons in the isolated atoms go together to form bands of levels, and that electrons in these bands can move freely through the completed crystal. The bands in an insulator such as ZnS are separated by gaps of energy in which lie no levels for electrons [Fig. 11(*a*)]. When there is no excitation, all the bands of energy levels are filled with electrons up through a highest filled band, and the bands above this are all unfilled. If a Cu atom happens to be included in the lot which is being brought together to form the ZnS crystal, it may turn out that the energy levels of this single stranger atom will not fall in with the levels of the Zn and S atoms which

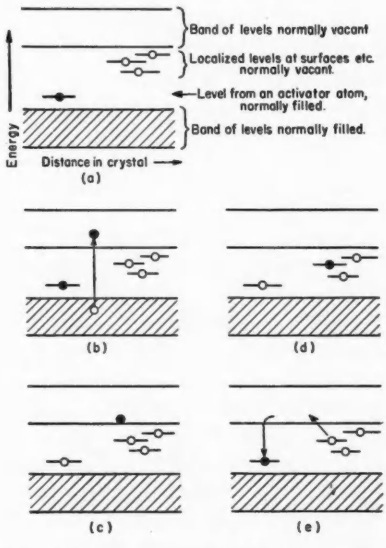

FIG. 11. (*a*) Distribution of energy levels for electrons in an impure insulating crystal such as ZnS·Cu. Excitation raises an electron from the lower filled band to a level in the upper band (*b*). Its place is taken by an electron from an activator level (*c*). The excited electron is trapped in a level associated with a lattice irregularity (*d*) and returns to the activator level, with emission of light, only after it has been re-excited to the upper band (*e*).

[18] For a nonmathematical discussion of this theory see F. Seitz and R. P. Johnson, J. App. Phys. **8**, 84, 186, 246 (1937). The application to phosphors is treated more fully by R. P. Johnson, J. Opt. Soc. Am. **29**, 387 (1939) and by N. Riehl and M. Schön, Zeits. f. Physik **114**, 682 (1939), among others.

make up the energy bands. In Fig. 11(a) is shown one such level belonging to an activator atom, which happens to fall in the gap of energy normally forbidden to electrons. This level is shown occupied by an electron, which is not free to move in the crystal but is bound to the neighborhood of the activator atom. Stranger atoms are not the only possible source of accessible energy levels in the normally forbidden gaps. The theory shows that wherever the regularity of the lattice is interrupted, at any surfaces or cracks or strains,[19] such levels exist. Fig. 11(a) shows several such levels, not normally occupied by electrons. An electron chancing to arrive in one of these levels would be unable to travel through the crystal, but would be retained very near the surface or the center of strain.

Figure 11(a), then, represents where electrons are and where they may go, in a small strained impure crystal such as ZnS·Cu. It is a crude picture, subject certainly to many improvements, but it is believed to be a simplification of the true state of affairs, and not an utter perversion.

The succeeding sketches show the life history of a single excitation, resulting finally in the emission of a quantum of the characteristic fluorescence light. An electron of the sulfide lattice is raised, by absorption of a quantum of ultraviolet radiation or by the impact of a bombarding electron or α-particle, from its normal place in the highest of the filled bands to a level in the next higher band [Fig. 11(b)]. This leaves a migratory vacancy in the lower band, into which falls the electron from one of the activator atoms [Fig. 11(c)]. The excited electron in its motion to and fro in the crystal may encounter this activator atom which has lost an electron; if so, it will drop down into this vacated level and a quantum will be emitted. But it may also happen—to explain the fact of phosphorescence it must be supposed very likely to happen—that the electron will first encounter a surface, a crack or a strained place, and will be trapped in one of the energy levels associated with such irregularities [Fig. 11(d)]. In this event it must wait until it has been freed by temperature excitation or other means, before it can again start out on a trip which may bring it to an activator atom with a vacant lower level. This final return, whenever it occurs, results in emission of a fluorescence quantum and restores the original situation in the crystal [Fig. 11(e)].

This account is not proposed for uncritical acceptance, but it is interesting to see how it fits with the observations on sulfides, which we have already reviewed. The activator atom must furnish a normally occupied energy level lying in a particular gap in the banded energy spectrum of the pure matrix material—this will happen only by favorable accident, and it is therefore reasonable that only a certain few combinations of base materials and activators are good phosphors. The color of the emitted light will depend on the position of this isolated energy level with respect to the bands; the spectral distribution should consequently vary with the activator, with the base material and with the physical state of the substance. The shape of the decay curve and its variation with temperature will depend on how many levels are available for trapping electrons and on how these levels are located below the upper band, which an electron must reach before it is free to wander. These levels, arising as they do from random irregularities, ought to vary markedly from sample to sample, and the decay curves should take correspondingly diverse courses. The rate at which trapped electrons are freed and allowed to seek vacated activator levels will be greater, the greater the thermal energy at large in the lattice—here is a natural explanation for the increased decay rate at higher temperatures and for the freezing-in of the light (that is, the freezing of trapped electrons in their places) at low temperatures.

Many other experimental observations support this general picture. It must be emphasized, however, that the model is far from being quantitative, that some of the assumptions it involves are arbitrary, and that in its simple form it necessarily leaves unexplained many aspects of the behavior of sulfide phosphors. By means of one or two *ad hoc* additions this rough model can be adapted to fit the silicates, with their initial exponential component of decay.

[19] Evidence from many fields indicated that such irregularities exist even in the most nearly perfect real crystals. A striking demonstration of their presence in single crystals of NaCl is given by Rexer, Zeits. f. Physik **76**, 735 (1932).

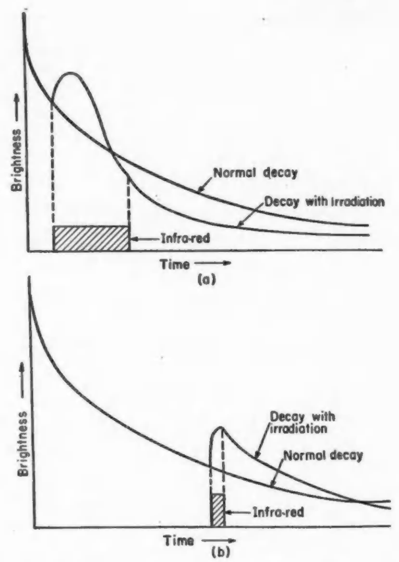

FIG. 12. (*a*) Continued exposure to low energy quanta reduces the supply of trapped electrons in a phosphorescing sulfide below its normal value, and the sample afterwards is abnormally dim. (*b*) Brief exposure to low energy quanta increases the number of trapped electrons in higher levels from which they can easily be freed by temperature excitation, and the sample afterwards is temporarily abnormally bright.

It is not yet clear how it should be modified to account for the distinctive properties of tungstate phosphors.

Low Energy Quanta and Excited Sulfides

The simple theory of sulfide phosphors sketched in the foregoing section is sufficient for explaining some effects with red and infra-red radiation, which make a rather entertaining demonstration. It is interesting to note that the influence of this kind of radiation on phosphors was first discovered by the German poet Goethe.

The radiation from a ruby photographic safe-lamp, as we saw earlier, is totally ineffective in exciting a sulfide. But suppose the sulfide has been excited, and is phosphorescing. We imagine many electrons trapped in irregularity-levels, with an occasional one being liberated by temperature excitation and either reverting to another irregularity-level, or finding its way to a vacant activator level and emitting a quantum. The energy necessary to free a trapped electron is relatively small—it is approximately the energy contained in a quantum of red or infra-red radiation. If the phosphorescing sample is exposed to the ruby lamp, the trapped electrons should be freed more rapidly and the decay should be hastened. It is hard to tell, without additional apparatus, whether the sample actually grows brighter during the exposure, since the light from the lamp itself is comparatively bright. But the brightness after the irradiation is stopped can be used as a test. If one-half of a phosphorescing sulfide screen is covered while the other half is exposed for a minute or two to the ruby lamp, afterwards when the two are compared in the dark it is seen that the irradiated half, which has been losing its trapped electrons rapidly, is much *less bright* than the other half, which still has a large supply of trapped electrons to tap [Fig. 12(*a*)].

The variant of this effect employs a sulfide sample that has been decaying for a considerable time and has reached a low level of brightness. According to the picture, most of the electrons that were trapped in levels close to the upper band have already been freed and have returned finally to activator levels. There remain chiefly the electrons trapped in low lying levels, requiring so much energy for their release that the temperature agitation can only infrequently free one of them. Suppose this sample is given a brief exposure to the ruby lamp. The radiation will raise many of the trapped electrons up to the freedom of the upper band. Some of these freed electrons will reach their final destinations in activator levels, but most of them will be trapped again en route. In this new trapping, however, they will not fall preferentially into the low lying levels from which they came, but will on the average go into levels much closer to the upper band. From these higher levels they can more easily be raised to the band and to freedom by temperature excitation. If one-half of the sample is exposed to the ruby lamp, very briefly, and then immediately afterwards is compared with the unexposed half, it is clear that the irradiation has markedly *increased* the brightness [Fig. 12(*b*)]. Here is an outcome apparently exactly opposite to that of the previous experiment, but the simple theory shows what changes in the experimental procedure are responsible for the different result.

The Collision of Two Particles

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THE general theory of the collision of two particles receives less attention in most of our mechanics textbooks than do many other topics of similar interest. Special applications are, of course, treated: the ballistic pendulum is treated in nearly all books; the interaction of the baseball and bat, and the bouncing of the cricket ball are given by Prescott;[1] the general motion of a billiard ball is given by Loney;[2] the theory of the pile driver is presented by the Grays;[3] additional interesting special cases are given by still other authors. But the more general equations, of which the foregoing examples are but special applications, are, as a rule, not included. However, the present-day importance of atomic and nuclear physics lends interest to single collision theory.

We shall denote the mass of one of the particles by m, its velocity before the impact by $\mathbf{a}$ and its velocity after the impact by $\mathbf{u}$; the corresponding quantities for the other particle are n, $\mathbf{b}$ and $\mathbf{v}$. We shall denote the sum of the momentums of the particles by $\mathbf{M}$, so that the law of conservation of momentum gives us

$$\mathbf{M}=m\mathbf{a}+n\mathbf{b}=m\mathbf{u}+n\mathbf{v}. \tag{1}$$

We shall let E_0 stand for the sum of the translational kinetic energies before impact, and E for the sum after, so that

$$\begin{aligned}2E_0&=ma^2+nb^2,\\ 2E&=mu^2+nv^2.\end{aligned} \tag{2}$$

Now, when $E=E_0$, the collision is said to be *elastic*, and when $E\neq E_0$, the collision is called *inelastic*. We shall denote the difference E_0-E by the letter Q. When Q is positive, translational kinetic energy is lost, it being converted into other forms, such as heat, noise or rotational kinetic energy; such a collision is called an *inelastic collision of the first kind*, or simply, an *inelastic collision*. When Q is negative, some other form of energy is transformed into translational kinetic energy; such a collision is called an *inelastic collision of the second kind*. Therefore, for an elastic collision, we have

$$ma^2+nb^2=mu^2+nv^2$$

and, for an inelastic collision,

$$ma^2+nb^2=mu^2+nv^2+2Q.$$

It is frequently helpful to consider the collision from the viewpoint of one who is moving with the center of mass of the two particles. It is easy to show that $\mathbf{V}$, the velocity of the center of mass, is given by

$$\mathbf{V}=\mathbf{M}/(m+n). \tag{3}$$

Quantities observed in the center-of-mass coordinates will be denoted by the subscript g. It follows easily that

$$E_g=E-M^2/2(m+n). \tag{4}$$

Now, the most elegant way to present this material is first to solve the equations for the center of mass system, and then to add the motion of the observer to the result obtained. However, the object of this paper is to present the material in such a way that the reader may select what he wishes for a particular purpose, without having to wade through the rest. We shall therefore begin with the simplest theory, and then proceed to more difficult topics. The material will be illustrated with problems that do not, as yet, appear in the mechanics textbooks.

(a) Collinear Elastic Impact

In this case, since all the vector quantities mentioned are collinear, we can replace them by their scalar magnitudes. Then the equations to be solved become

$$ma+nb=mu+nv, \tag{1a}$$

$$ma^2+nb^2=mu^2+nv^2. \tag{2a}$$

These are simultaneous equations in the two

[1] Prescott, *Mechanics of particles and of rigid bodies* (Longmans, Green, ed. 2, 1923), pp. 476 ff.

[2] Loney, *Elementary treatise on the dynamics of particles and of rigid bodies* (Cambridge, 1923), p. 302.

[3] A. Gray and J. G. Gray, *Treatise on dynamics* (Macmillan, 1931), pp. 395 ff.

unknowns, u and v. Transposing them and dividing one by the other, we have

$$a+u=v+b,$$

or, as it is more usually written,

$$(a-b)=-(u-v). \tag{5a}$$

We have thus reduced the pair of simultaneous Eqs. (1a) and (2a) to the pair (1a) and (5a), both of them linear. The solutions are

$$u=[2nb+(m-n)a]/(m+n), \tag{6a}$$

$$v=[2ma+(n-m)b]/(m+n). \tag{7a}$$

If $m=n$, then $u=b$ and $v=a$, or the particles merely exchange velocities. If n be infinite, we have the normal incidence of an elastic sphere of mass m on a table or wall moving with velocity b, giving $u=2b-a$ and $v=b$. The fascinating case for which $n=3m$ has already been treated in this journal.[4]

Equation (5a) is of interest as it is independent of the masses. It simply says that, in a collinear elastic collision, the velocity of separation after the impact is equal in magnitude but opposite in direction to the velocity of approach before the impact.

Collinear elastic impact is really quite simple, and is suitable for inclusion in the first-year college course.[5]

(b) Collinear Inelastic Impact

Consider first the quantity E_0 in Eq. (4). Denote by e^2 the ratio of this kinetic energy after impact to that before; thus

$$e^2=\frac{2(m+n)E-M^2}{2(m+n)E_0-M^2}. \tag{8}$$

Substituting Eqs. (1a) and (2) in this, and extracting the square root, we obtain

$$e(a-b)=\pm(u-v), \tag{5b}$$

where, in general, only the negative sign is of interest, and where e can have all values from zero to infinity. When $e=0$, the two particles move off together after the impact; one "captures" the other, as in the case of the ballistic pendulum or the coupling of railroad cars. When $0<e<1$ ordinary inelastic collision of the first kind results. When $e=1$ we have Eq. (5a) and the elastic collision already discussed. When $e>1$ we have an inelastic collision of the second kind. When e is very large we have such phenomena as the firing of a gun or radioactive disintegration. These may not be "collisions" exactly, but they are at least limiting cases.

For an instance in which the positive sign in Eq. (5b) applies, consider a ballistic pendulum whose bob is too short, so that the bullet emerges from the far end with some momentum. This would be an inelastic collision in which the relative velocity after impact is in the same direction as before.

It was shown by Newton[6] that, for many substances, e is independent of the speed of impact over a wide range, but depends only upon the nature of the surfaces at the point of contact. Under these circumstances, e is regarded as a property of the material composing the particles, is called the *coefficient of restitution* and can have values only between zero and unity.

Combining Eqs. (5b) and (1a), we obtain

$$u=\frac{n(1+e)b+(m-en)a}{(m+n)}, \tag{6b}$$

$$v=\frac{m(1+e)a+(n-em)b}{(m+n)}. \tag{7b}$$

From Eq. (8), we get

$$\left.\begin{aligned} E&=e^2\left[E_0-\frac{M^2}{2(m+n)}\right]+\frac{M^2}{2(m+n)},\\ &=\frac{e^2mn(a-b)^2+(ma+nb)^2}{2(m+n)},\\ Q&=E_0-E=(1-e^2)\left[E_0-\frac{M^2}{2(m+n)}\right]\\ &=\frac{mn(a-b)^2(1-e^2)}{2(m+n)}.\end{aligned}\right\} \tag{9}$$

[4] H. B. Lemon, Am. J. Phys. (Am. Phys. T.) **3**, 36 (1935).

[5] A few of the elementary books do include some of this material, either in the text or as a problem at the end of a chapter; for example, Weld and Palmer, *A textbook of modern physics* (Blakiston, ed. 2, 1930), pp. 152 ff., esp. prob. 11, p. 155.

[6] Jeans, *Elementary treatise on theoretical mechanics* (Ginn, 1907), pp. 241–248.

Thus, if e is known, we can compute u, v and E or Q from Eqs. (6b), (7b) and (9); whereas, if E is known, we can find e and then u and v from Eqs. (8), (6b) and (7b).[7]

(c) Most General Statement of Collinear Impact

Suppose that we wish to express u and v in terms of m, n, M and E. Our given conditions are

$$M = mu + nv, \tag{1c}$$

$$2E = mu^2 + nv^2. \tag{2c}$$

Elimination of v from these equations gives

$$m(m+n)u^2 - 2mMu - 2nE + M^2 = 0, \tag{14c}$$

the solution of which is

$$u = \frac{mM \pm [2mn(m+n)E - mnM^2]^{\frac{1}{2}}}{m(m+n)}. \tag{6c}$$

Similarly, for v there results

$$v = \frac{nM \mp [2mn(m+n)E - mnM^2]^{\frac{1}{2}}}{n(m+n)}, \tag{7c}$$

where the two upper signs go together. The significance of the two solutions can be seen by referring to Eqs. (1a) and (2a), where $E = E_0$: if, in an elastic impact, the upper pair of signs in Eqs. (6c) and (7c) represent the velocities after the impact, the lower pair give the expressions for a and b, and vice versa.

Suppose we have a collision wherein the particles exchange portions of their masses, as in an atomic disintegration, or in the well-known demonstration consisting of several balls, each suspended by two cords from a horizontal support. Then, by substituting the given initial conditions for M and E in Eqs. (6c) and (7c), we can find the values of u and v for the final conditions. Indeed, Eqs. (6c) and (7c) give us the solution for any possible type of collinear impact.

In atomic and nuclear physics, it is frequently more useful or convenient to know the kinetic energies of the particles rather than their velocities or momentums. Calling the kinetic energy of one of the particles E_m, where $E_m = \frac{1}{2}mu^2$, and using this relation to remove u from Eq. (14c), we easily obtain

$$E_m = \frac{2mM^2 + (m+n)(2nE - M^2) \pm 2M[m^2M^2 + m(m+n)(2nE - M^2)]^{\frac{1}{2}}}{2(m+n)^2} \tag{6c'}$$

and a similar expression for E_n, the kinetic energy of the other particle.

(d) Oblique Elastic Collision, Particle n Initially at Rest

Although this is merely a special case of the problem to be treated in Sec. (e), it is the case most frequently observed in real life, and so merits separate consideration. The equations to be solved are

$$m\mathbf{a} = m\mathbf{u} + n\mathbf{v}, \tag{1d}$$

$$ma^2 = mu^2 + nv^2. \tag{2d}$$

Here, as will be shown later, we have three simultaneous equations in four unknowns. Let

$$\begin{aligned} \mathbf{u} &= p\mathbf{a} + q\mathbf{j}, \\ \mathbf{v} &= r\mathbf{a} - s\mathbf{j} + t\mathbf{k}, \end{aligned} \tag{10d}$$

where $\mathbf{j}$ is any vector perpendicular to $\mathbf{a}$, and $\mathbf{k}$ is the vector perpendicular to $\mathbf{a}$ and $\mathbf{j}$. Since $\mathbf{j}$ and $\mathbf{k}$ can be given any magnitude we please, we shall find it convenient to give them the same magnitude as $\mathbf{a}$, so that

$$\begin{aligned} \mathbf{a}\cdot\mathbf{a} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = a^2, \\ \mathbf{a}\cdot\mathbf{j} = \mathbf{a}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{j} = 0. \end{aligned}$$

This is equivalent to erecting an $\mathbf{ijk}$ system of vectors at the point of impact, where $\mathbf{i}$ coincides with $\mathbf{a}$, and where the coordinate vectors are given the magnitude a instead of unity.

Substitution of Eqs. (10d) in Eq. (1d) yields

$$\begin{aligned} mp + nr &= m, \\ mq - ns &= 0, \\ nt &= 0. \end{aligned} \tag{11d}$$

From the equation, $t = 0$, we see that the vectors $\mathbf{a}$, $\mathbf{u}$ and $\mathbf{v}$ must be coplanar. Then, by referring to Fig. 1, we see that the coefficients p, q, r and

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[7] For other derivations of Eqs. (6b), (7b) and (9) or their equivalents, see: Frank, *Mechanics and heat* (McGraw-Hill, 1934), pp. 86 ff.; Jeans, reference 6; Lindsay, *Physical mechanics* (Van Nostrand, 1933), p. 350; Millikan, Roller and Watson, *Mechanics, molecular physics, heat and sound* (Ginn, 1937), pp. 90–107.

For laboratory exercises using this material, see Millikan *et al.*, and Wadlund, Am. J. Phys. (Am. Phys. T.) 7, 164 (1939).

FIG. 1.

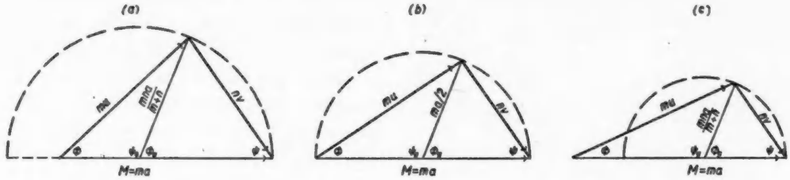

FIG. 2.

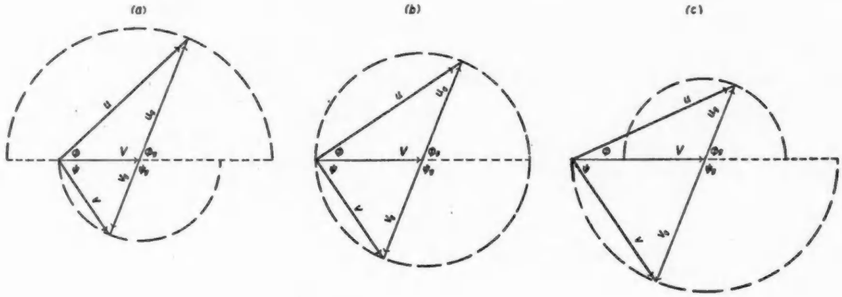

FIG. 3.

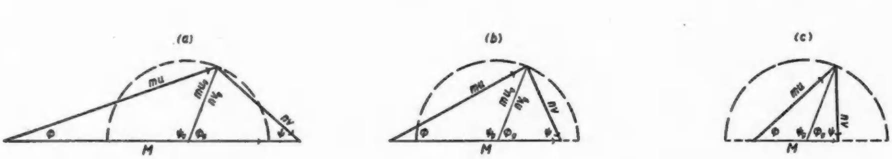

FIG. 4.

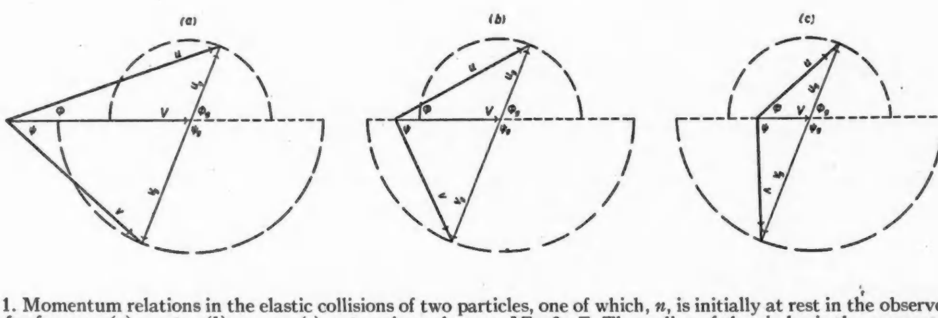

FIG. 1. Momentum relations in the elastic collisions of two particles, one of which, n, is initially at rest in the observer's frame of reference: (a), $m<n$; (b), $m=n$; (c), $m>n$; in each case, $M^2=2mE$. The radius of the circles is the momentum of each particle as observed in the center-of-mass coordinate system. ϕ_g and ψ_g are the angles between the initial and final directions of motion as observed in the center-of-mass system; ϕ and ψ are the angles of scattering and recoil, respectively, in the observer's system. The mass m and the velocity a of the impinging particle are taken as unity. The vector triangles represent the relation $\mathbf{M}=m\mathbf{a}=m\mathbf{u}+n\mathbf{v}=$constant.

FIG. 2. Velocity relations in the same collisions as those depicted in Fig. 1. Here, the radius of the circles is the velocity of the particles as seen in the center-of-mass coordinates. These figures are the polar graphs of the simultaneous equations (6d), (7d) and (15d) for different values of the ratio m/n. Fig. 1 is also a graphical representation of these same equations.

FIG. 3. Momentum relations in collisions in which either particle may have any momentum whatever before the impact: (a) $M^2>2mE$; (b) $2mE>M^2>2nE$; (c) $2nE>M^2$; in each case, $m>n$. If we confine our attention to elastic collisions, then these diagrams represent the collisions of Fig. 1(c) as seen by three other observers (than the one for whom $M^2=2mE$). In any event, ϕ_g and ψ_g are the angles between the final directions of motion of the particles (as observed in the center-of-mass system) and the motion of the center of mass with respect to the other observers; ϕ and ψ are the angles between the motions of the particles (with respect to the observer's frame) and $\mathbf{M}$.

FIG. 4. Velocity relations in the collisions shown in Fig. 3. These are the polar graphs of Eqs. (6e) and (7e) [which are connected by Eqs. (15e)] for various values of E and M. Fig. 3 is also a graphical representation of these equations.

s have the following significances:

$$p=u\cos\phi/a, \qquad q=u\sin\phi/a, \qquad r=v\cos\psi/a, \qquad s=v\sin\psi/a. \tag{12d}$$

Thus, Eqs. (11d) can be rewritten as follows:

$$ma=mu\cos\phi+nv\cos\psi, \qquad 0=mu\sin\phi-nv\sin\psi, \tag{1d'}$$

from which it is seen that Eq. (1d) is implicitly two simultaneous equations with four unknowns, u, v, ϕ and ψ.

Substituting Eqs. (10d) in (2d), we have

$$m(p^2+q^2)+n(r^2+s^2)=m. \tag{13d}$$

Eliminating r and s from this by means of Eqs. (11d) and substituting Eqs. (12d) in the result, we obtain

$$(m+n)(u/a)^2-2m(u/a)\cos\phi=n-m, \tag{14d}$$

the solution to which is

$$u=\frac{a[m\cos\phi\pm(n^2-m^2\sin^2\phi)^{\frac{1}{2}}]}{m+n}. \tag{6d}$$

Similarly, elimination of p and q from Eq. (13d) yields

$$v=2am\cos\psi/(m+n). \tag{7d}$$

Eq. (6d) gives the speed of the scattered particle as a function of the scattering angle ϕ; Eq. (7d) gives the speed of the recoil particle as a function of the angle of recoil ψ. However, ϕ and ψ are not independent. We need, therefore, expressions giving us one in terms of the other. There are several of these, the ones frequently quoted being

$$\tan\phi=n\sin 2\psi/(m-n\cos 2\psi), \qquad \tan\psi=\frac{n\cot\phi\pm(n^2\operatorname{cosec}^2\phi-m^2)^{\frac{1}{2}}}{m+n}. \tag{15d}$$

The derivation is given in Sec. (f).

Equation (7d) is the polar equation of the circle passing through the origin with radius $ma/(m+n)$, and having its center at $[ma/(m+n), 0]$; Eq. (6d) is the equation of the circle concentric with that of Eq. (7d), but having radius $na/(m+n)$. These relations are depicted in Fig. 2. From Fig. 2 (or by considering the cases for which $\phi=0$ and $\phi=\pi$, without recourse to the figure), we see that the negative sign has physical significance only when m is greater than n. If m is greater than n, then for each scattering angle ϕ there are two scattering velocities u, given by Eq. (6d), and two recoil angles ψ given by the second of Eqs. (15d); and to the value of ψ so obtained, there is only one recoil velocity v, given by Eq. (7d).

When $m=n$, $u=a\cos\phi$ and $v=a\cos\psi$. Substitution of these in Eq. (1d′) yields

$$\cos^2\phi+\cos^2\psi=1, \qquad \phi+\psi=\pi/2.$$

Passage of neutrons through matter.—When neutrons pass through the cloud chamber, the only measurable quantities are the track of the recoil nucleus and the assumed angle of recoil. If the range-energy relation of the recoil nucleus is known, the energy of the incident neutron can be found. To simplify things a bit, let the ratio $n/m=A$, where A is the atomic weight of the recoil nucleus. Then Eqs. (6d) and (7d) become

$$u=a[\cos\phi+(A^2-\sin^2\phi)^{\frac{1}{2}}]/(A+1), \qquad v=2a\cos\psi/(A+1).$$

This latter equation may be rewritten as

$$a=\tfrac{1}{2}(A+1)v\sec\psi$$

or, in terms of the energies of the particles,

$$E_0=(1/4A)(A+1)^2E_n\sec^2\psi$$

where E_0 and E_n are the kinetic energies of the incident neutron and of the recoil nucleus, respectively. Of course, this applies only if the collision is elastic, but there is no way to tell whether a given collision is elastic or not.

When a beam of neutrons passes through a block of material, the phenomenon of interest in certain experiments is the slowing of the neutrons by elastic impacts with the nuclei composing the material. Now, in an elastic collision, the kinetic energy lost by the incident particle is gained by the recoil nucleus. We seek an expression for the average fractional loss of kinetic energy suffered by the neutrons in a single collision. Denoting this fractional loss by ω, we have

$$\omega=\frac{\Delta E_0}{E_0}=\frac{E_n}{E_0}=\frac{Av^2}{a^2}=\frac{4A\cos^2\psi}{(A+1)^2}$$

$$\omega_{\text{Av}}=\frac{4A}{(A+1)^2}(\cos^2\psi)_{\text{Av}}.$$

To determine $(\cos^2\psi)_{\text{Av}}$ is a problem of the quantum mechanics. In the case of s scattering $(\cos^2\psi)_{\text{Av}}=\frac{1}{2}$, so that $\omega_{\text{Av}}=2A/(A+1)^2$. The classical analog of s scattering is treated in Sec. (f). When $A=1$ (passage of neutrons through hydrogenous material, such as water or paraffin), $\omega_{\text{Av}}=\frac{1}{2}$; when A is very large, $\omega_{\text{Av}}=2/A$. Then, if the average number of collisions each neutron will have in passing through a given thickness of a given material can be estimated, the slowing effect of the arrangement at hand can also be estimated. This is the method used by Hornbostel and Valente[8] to resolve the nuclear energy levels of I^{127}.

[8] Hornbostel and Valente. Phys. Rev. 55. 108 (1939).

(e) Most General Type of Collision

Suppose we are given m, n, E and $\mathbf{M}$, and wish to find $\mathbf{u}$ and $\mathbf{v}$. The equations to be solved are

$$\mathbf{M}=m\mathbf{u}+n\mathbf{v}, \tag{1e}$$

$$2E=mu^2+nv^2. \tag{2e}$$

Resolution of $\mathbf{u}$ and $\mathbf{v}$ into rectangular components gives

$$\begin{aligned}\mathbf{u}&=p\mathbf{M}+q\mathbf{j},\\ \mathbf{v}&=r\mathbf{M}-s\mathbf{j}+t\mathbf{k},\end{aligned} \tag{10e}$$

where, in this case, $\mathbf{j}$ and $\mathbf{k}$ are vectors at right angles to $\mathbf{M}$, and of magnitude M. Substituting Eqs. (10e) in (1e), we get

$$\begin{aligned}mp+nr&=1,\\ mq-ns&=0,\\ nt&=0,\end{aligned} \tag{11e}$$

showing that $\mathbf{M}$, $\mathbf{u}$ and $\mathbf{v}$ are coplanar; but $\mathbf{u}$ and $\mathbf{v}$ need not be coplanar with $\mathbf{a}$ and $\mathbf{b}$—they can be rotated at random about the vector $\mathbf{M}$. By referring to Fig. 3, we see that

$$\begin{aligned}p&=u\cos\phi/M, & q&=u\sin\phi/M,\\ r&=v\cos\psi/M, & s&=v\sin\psi/M,\end{aligned} \tag{12e}$$

so that we can rewrite Eqs. (11e) as

$$\begin{aligned}M&=mu\cos\phi+nv\cos\psi,\\ 0&=mu\sin\phi-nv\sin\psi.\end{aligned} \tag{1e$'$}$$

Substitution of Eqs. (10e) in (2e) yields

$$[m(p^2+q^2)+n(r^2+s^2)]M^2=2E. \tag{13e}$$

Eliminating r and s from this by means of Eqs. (11e), and substituting (12e) in the result, we obtain

$$m(m+n)u^2-2mMu\cos\phi=2nE-M^2, \tag{14e}$$

the solution to which is

$$u=\frac{mM\cos\phi\pm\{m[n(m+n)2E-M^2(n+m\sin^2\phi)]\}^{\frac{1}{2}}}{m(m+n)}. \tag{6e}$$

Similarly, by eliminating p and q from Eq. (13e), we obtain

$$v=\frac{nM\cos\psi\mp\{n[m(m+n)2E-M^2(m+n\sin^2\psi)]\}^{\frac{1}{2}}}{n(m+n)}. \tag{7e}$$

Also, from Eq. (14e), we may obtain the expression for the kinetic energy of one of the particles,

$$E_m=\frac{2mM^2\cos^2\phi+(m+n)(2nE-M^2)\pm2M[m^2M^2\cos^4\phi+m(m+n)(2nE-M^2)\cos^2\phi]^{\frac{1}{2}}}{2(m+n)^2}$$

and a similar expression for E_n.

Equation (6e) is the polar equation of a circle of radius

$$\{nm[2(m+n)E-M^2]\}^{\frac{1}{2}}/m(m+n)$$

and center at $[M/(m+n), 0]$; Eq. (7e) is the polar equation of the circle concentric with this one, but having the radius

$$\{mn[2(m+n)E-M^2]\}^{\frac{1}{2}}/n(m+n).$$

These relations are depicted in Fig. 4. The equations connecting ϕ and ψ are

$$\begin{aligned}\tan\phi&=\frac{2mnE\cot\psi\mp\{(2mnE\cot\psi)^2+(2mnE-nM^2)[(m+n)M^2-(2mnE-nM^2)\cot^2\psi]\}^{\frac{1}{2}}}{(m+n)M^2-(2mnE-nM^2)\cot^2\psi},\\ \tan\psi&=\frac{2mnE\cot\phi\pm\{(2mnE\cot\phi)^2+(2mnE-mM^2)[(m+n)M^2-(2mnE-mM^2)\cot^2\phi]\}^{\frac{1}{2}}}{(m+n)M^2-(2mnE-mM^2)\cot^2\phi}.\end{aligned} \tag{15e}$$

They are derived in Sec. (f).

The question of which sign to use can easily be answered by referring to the figures. In order that the negative sign may have physical significance, the origin must lie outside the circle—there must be two possible values for the velocity concerned, and the negative sign gives the smaller velocity in Eqs. (6e) and (7e). To the larger of the two velocities possible at a given angle ϕ, there corresponds the smaller of the two velocities which are possible at the angle ψ given by the + sign in the second of Eqs. (15e).

Transmutation of the atomic nucleus.—Perhaps the most important application of Eqs. (6e) and (7e) is the bombardment of the nucleus. Before the disintegration there are two particles (the more massive one generally at rest with respect to the observer); after the collision there are two different particles, but, in general, the motion of only the lighter is measurable. Such collisions are generally very inelastic, the gain or loss of translational kinetic energy being one of the most important items to try to ascertain from the experiments, as it is from this that atomic masses and binding energies may be obtained. Calling the mass of the lighter particle before the impact (the "projectile") m_0, that of the target nucleus, n_0, that of the "emitted" particle, m and that of the residual nucleus, n, we have, apart from relativistic considerations, which can usually be neglected,[9]

E_0 = translational kinetic energy of projectile,
M = momentum of projectile $=\sqrt{(2m_0E_0)}$
= sum of momentums of emitted particle and residual nucleus after impact,
E_m = kinetic energy of emitted particle $=\frac{1}{2}mu^2$,
E = sum of kinetic energies of emitted particle and residual nucleus.

Substituting these quantities in Eq. (6e), or, better still, directly in Eq. (14e), we have

$$2(m+n)E_m-4(mE_m m_0E_0)^{\frac{1}{2}}\cos\phi=2(nE-m_0E_0).$$

Solving for E, we obtain

$$E=\frac{(m+n)E_m+m_0E_0-2(mE_m m_0E_0)^{\frac{1}{2}}\cos\phi}{n}$$

$$Q=E_0-E=\frac{2(mE_m m_0E_0)^{\frac{1}{2}}\cos\phi-(m+n)E_m+(n-m_0)E_0}{n}.$$

(f) Center-of-mass Coordinate System

If $\mathbf{M}=0$, the center of mass of the two particles is at rest with respect to the observer, and Eqs. (6e) and (7e) become

$$u_g=\frac{[2mn(m+n)E_g]^{\frac{1}{2}}}{m(m+n)},\qquad(6f)$$

$$v_g=\frac{[2mn(m+n)E_g]^{\frac{1}{2}}}{n(m+n)}.\qquad(7f)$$

Eqs. (6f) and (7f) are the polar equations of the same circles as those in Eqs. (6e) and (7e), but here referred to their own centers as origins. They are the solutions of the equations

$$0=mu_g+nv_g,\qquad(1f)$$

$$2E_g=mu_g^2+nv_g^2.\qquad(2f)$$

Now, as remarked before, it is possible to start with Eqs. (1f) and (2f), and then, by adding the velocity of the observer to the solutions [Eqs. (6f) and (7f)], to obtain all of the foregoing material. Thus, for the elastic collision,

$$u_g=a_g,\qquad v_g=b_g;$$

for the inelastic collision,

$$u_g=ea_g,\qquad v_g=eb_g,$$

where

$$a_g=\frac{[2mn(m+n)E_{0g}]^{\frac{1}{2}}}{m(m+n)},$$

$$b_g=\frac{[2mn(m+n)E_{0g}]^{\frac{1}{2}}}{n(m+n)}.$$

Translating to any other Galilean coordinate system, we have

$$\mathbf{a}=\mathbf{V}+\mathbf{a}_g,\qquad \mathbf{b}=\mathbf{V}+\mathbf{b}_g,$$
$$\mathbf{u}=\mathbf{V}+\mathbf{u}_g,\qquad \mathbf{v}=\mathbf{V}+\mathbf{v}_g.$$

Combining these with Eqs. (3), (4) and (8), we can derive all of the preceding equations. This is probably the best way to present the subject in mechanics; the other methods are suitable if one wishes to present only one specific set of conditions without going into the general solution.

Derivation of Eqs. (15d) and (15e).—Having derived Eqs. (6f) and (7f), we shall now illustrate the use of the center-of-mass coordinate system by obtaining Eqs. (15d) and (15e) simply by transposing from this system to any other system.

The first of Eqs. (15d), however, can be more easily obtained merely by transposing the first two of Eqs. (11d)

[9] Bethe and Livingston, Rev. Mod. Phys. 9, 276 (1937).

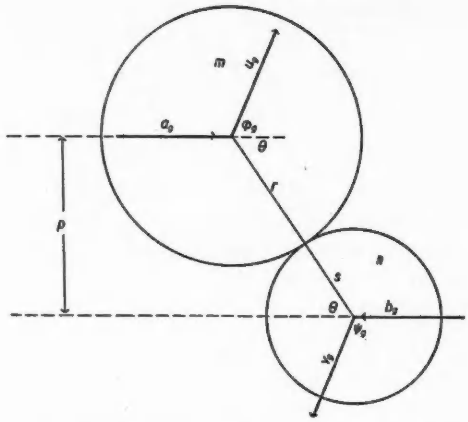

FIG. 5. Smooth elastic collision of two spheres of any size, mass or density whatever, as seen in the center-of-mass coordinate system.

and dividing one by the other, giving

$$q/p=\tan\phi=ns/(m-nr)=2n\sin\psi\cos\psi/(m-2n\cos^2\psi)$$
$$=n\sin 2\psi/(m-n\cos 2\psi).$$

The easiest way to derive the rest of the relations just mentioned is as follows. Referring to Figs. 1 or 2, we see that

$$\tan\phi_0=-\tan\psi_0,\qquad \csc\phi_0=\csc\psi_0,$$
$$v_0=V=b_0=ma/(m+n),\qquad u_0=a_0=na/(m+n),$$
$$v_x=V+v_{0x},\qquad u_x=V+u_{0x},$$
$$v_y=v_{0y},\qquad u_y=u_{0y},$$
$$\cot\phi=\frac{u_x}{u_y}=\frac{u_{0x}}{u_{0y}}+\frac{V}{u_{0y}}=\cot\phi_0+(m/n)\csc\phi_0,$$
$$\cot\psi=\frac{v_x}{v_y}=\frac{v_{0x}}{v_{0y}}+\frac{V}{v_{0y}}=\cot\psi_0+\csc\psi_0=-\cot\phi_0+\csc\phi_0.$$

Adding and subtracting these two equations for $\cot\phi$ and $\cot\psi$, we have

$$\cot\phi+\cot\psi=(m+n)/n\csc\phi_0,$$
$$\cot\phi-(m/n)\cot\psi=(m+n)/n\cot\phi_0.$$

Squaring each of these and subtracting, we obtain an expression that easily reduces to

$$(n-m)\cot^2\psi+2n\cot\phi\cot\psi-(m+n)=0.$$

Multiplication of both members of this equation by $\tan^2\psi$, and solution of the resulting expression for $\tan\psi$ yields the second of Eqs. (15d).

For the more general case [Eqs. (15e)], referring to Figs. 3 or 4, using Eqs. (3), (6f), (7f) and (4), and following the procedure just outlined, we obtain

$$\cot\phi=\cot\phi_0+\frac{mM}{\{mn[2(m+n)E-M^2]\}^{\frac{1}{2}}}\csc\phi_0,$$
$$\cot\psi=-\cot\phi_0+\frac{nM}{\{mn[2(m+n)E-M^2]\}^{\frac{1}{2}}}\csc\phi_0.$$

Adding and subtracting these, we get

$$\{mn[2(m+n)E-M^2]\}^{\frac{1}{2}}(\cot\phi+\cot\psi)=(m+n)M\csc\phi_0,$$
$$M(n\cot\phi-m\cot\psi)=(m+n)M\cot\phi_0.$$

Squaring each of these, subtracting and reducing gives

$$(2mnE-nM^2)\cot^2\phi+4mnE\cot\phi\cot\psi$$
$$+(2mnE-mM^2)\cot^2\psi=(m+n)M^2.$$

Multiplication of both members of this by $\tan^2\phi$ and solution for $\tan\phi$ yields the first of Eqs. (15e); the method of obtaining the second is now obvious.

Any possible combination of velocities and angles in Eqs. (6e), (7e) and (15e) can just as easily represent conditions before the impact as after. If the collision is collinear in the center-of-mass system, the collision is called a central impact in any coordinate system, and Eqs. (6e) and (7e) reduce to the form of Eqs. (6a) and (7a):

$$\mathbf{u}=[2n\mathbf{b}-(m-n)\mathbf{a}]/(m+n),$$
$$\mathbf{v}=[2m\mathbf{a}+(n-m)\mathbf{b}]/(m+n).$$

Angular distribution of scattered and recoil particles in the collision in which one particle is initially at rest in the observer's coordinate system.—Referring to the paragraph on the passage of neutrons through matter in Section d, we now seek to justify the statement that, for the classical analog of quantum-mechanical s scattering, $\cos^2\psi_{\mathrm{Av}}=\frac{1}{2}$. Consider the oblique collision of two smooth elastic spheres in the center-of-mass system, illustrated in Fig. 5. Since the spheres are perfectly smooth (coefficient of friction $=0$), there can be no tangential force at the points of contact, so the entire impulse must be along the line joining their centers. Let this line make the angle θ with the initial direction of motion of the particles. Then

$$\theta=\tfrac{1}{2}(\pi-\phi_0)=\tfrac{1}{2}\psi_0.$$

But, from Fig. 2, $\psi=\frac{1}{2}\psi_0$, for the angle inscribed in a circle is one-half the central angle of same arc. Therefore $\theta=\psi$.

If p be the distance between the initial paths of the two particles, then $p=(r+s)\sin\theta$, where r is the radius of one sphere and s is the radius of the other. Observe that no conditions are imposed on the relative magnitudes of r and s. Let σ be the number of particles passing through unit area of the circle of radius $r+s$; let N be the total number (of mass m) impinging on the particle (of mass n) assumed to be at rest in the observer's frame of reference; and let dN be the number whose centers lie between p and $p+dp$. Then

$$dN=2\pi\sigma p dp=2\pi\sigma(r+s)^2\sin\theta\cos\theta d\theta$$
$$=2\pi\sigma(r+s)^2\sin\psi\cos\psi d\psi.$$

Now, if σ is constant (classical analog of s scattering), then $N=\pi\sigma(r+s)^2$, and the foregoing equation becomes

$$dN=2N\sin\psi\cos\psi d\psi.$$

Then the average value of $\cos^2\psi$ is given by

$$\cos^2\psi_{\mathrm{Av}}=\frac{\int_0^N\cos^2\psi dN}{N}=2\int_0^{\pi/2}\cos^3\psi\sin\psi d\psi=\tfrac{1}{2}.$$

Vector Analysis in Special Relativity

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Fundamental Turning Operators; Unit Vectors

It was pointed out in a previous paper[1] that any four-vector interval of proper time $d\tau$ can be most naturally expressed as

$$d\boldsymbol{\tau} = \mathbf{i}_0 dt + \mathbf{i}_1 dt^1 + \mathbf{i}_2 dt^2 + \mathbf{i}_3 dt^3, \tag{1}$$

where dt, dt^1, dt^2, dt^3 are the real time measures of the component displacements, where $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$ are turning operators producing the three space vectors from their time measures and where $\mathbf{i}_0$ is a unit vector in the direction of time flow. Since a double application of any of the three turning operators results in a negative time, the scheme of products adopted is

$$\begin{aligned} &\mathbf{i}_0\cdot\mathbf{i}_0 = 1, \quad \mathbf{i}_m\cdot\mathbf{i}_m = -1, \quad m = 1, 2, 3 \\ &\mathbf{i}_\mu\cdot\mathbf{i}_\nu = 0, \quad \mu \neq \nu, \ = 0, 1, 2, 3. \end{aligned} \tag{2}$$

The scalar product of any two intervals, obtained by applying Eqs. (2) and the distributive rule, is necessarily invariant for rotations of the base units which leave Eqs. (2) true.

The invariant scalar product of an interval with itself becomes

$$d\boldsymbol{\tau}\cdot d\boldsymbol{\tau} = d\tau^2 = dt^2 - (dt^1)^2 - (dt^2)^2 - (dt^3)^2, \tag{3}$$

and with the cgs system of units this gives the usual Minkowski interval,

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2. \tag{4}$$

Introduce the set of operators $\mathbf{i}^m$ *reciprocal* to $\mathbf{i}_m$; this means that $\mathbf{i}^m$ turns the time interval in the reverse sense to $\mathbf{i}_m$, or a successive operation $\mathbf{i}^m\cdot\mathbf{i}_m$ returns the space vector back to its original time direction, without change in magnitude. This successive operation is therefore equivalent to unity:

$$\mathbf{i}^m\cdot\mathbf{i}_m = 1, \qquad m = 1, 2, 3. \tag{5}$$

It is evident from a comparison of Eqs. (5) and (2) that

$$\mathbf{i}^m = -\mathbf{i}_m, \qquad m = 1, 2, 3, \tag{6}$$

and that, therefore,

$$\mathbf{i}^m\cdot\mathbf{i}_n = \delta_{mn}. \tag{7}$$

Eqs. (7) are usually adopted as the analytic definition of the reciprocal set; from them and Eqs. (2) we see that

$$\mathbf{i}^0 = \mathbf{i}_0. \tag{8}$$

By means of the reciprocal sets of operators any four-vector can be expressed in alternative ways,

$$\begin{aligned} \mathbf{A} &= A^0\mathbf{i}_0 + A^1\mathbf{i}_1 + A^2\mathbf{i}_2 + A^3\mathbf{i}_3 \\ &= A_0\mathbf{i}^0 + A_1\mathbf{i}^1 + A_2\mathbf{i}^2 + A_3\mathbf{i}^3, \end{aligned}$$

and the square length of the vector can be expressed as

$$(A)^2 \equiv \mathbf{A}\cdot\mathbf{A} = A_0A^0 + A_1A^1 + A_2A^2 + A_3A^3. \tag{9}$$

Because of Eq. (6), we have

$$A_m = -A^m, \tag{10}$$

and hence

$$(\mathbf{A})^2 = (A^0)^2 - (A^1)^2 - (A^2)^2 - (A^3)^2. \tag{11}$$

The form (9) permits use of the familiar dummy suffix rule,

$$(\mathbf{A})^2 = A_\mu A^\mu, \tag{12}$$

where a repeated Greek suffix implies summation over all four values of the suffix, 0, 1, 2, 3.

Dyadic Areas and Vector Volumes

In four dimensions an area must be represented by an antisymmetric dyadic containing the *two* directions normal to the area. Introduce the familiar symbols $e^{\alpha\beta\gamma\delta} = 0, +1, -1$, respectively, if any two of the suffixes are alike, if they form an even permutation of the set 0123, if they form an odd permutation. Retaining the cross-product notation, we shall write for the area enclosed by two unit vectors $\mathbf{i}^\alpha$, $\mathbf{i}^\beta$ the symbol

$$\mathbf{i}^\alpha \times \mathbf{i}^\beta \equiv e^{\alpha\beta\delta\gamma}\mathbf{i}_\gamma\mathbf{i}_\delta. \tag{13}$$

For example, the dyadic representing the area $\mathbf{i}^1$, $\mathbf{i}^2$ will then be

$$\mathbf{i}^1 \times \mathbf{i}^2 = \mathbf{i}_3\mathbf{i}_0 - \mathbf{i}_0\mathbf{i}_3. \tag{14}$$

[1] Am. J. Phys. (Am. Phys. Teacher) **6**, 323–324 (1938).

If such a dyadic operates upon another unit vector $\mathbf{i}^\mu$, the result is, from Eqs. (13) and (7),

$$\mathbf{i}^\alpha \times \mathbf{i}^\beta \cdot \mathbf{i}^\mu = e^{\alpha\beta\delta\gamma}\mathbf{i}_\gamma \mathbf{i}_\delta \cdot \mathbf{i}^\mu = e^{\alpha\beta\mu\delta}\mathbf{i}_\delta. \tag{15}$$

For example,

$$\mathbf{i}^0 \times \mathbf{i}^2 \cdot \mathbf{i}^3 = \mathbf{i}_1, \tag{16}$$

or the dyadic operator $\mathbf{i}^0 \times \mathbf{i}^2$ has the effect of changing the operator $\mathbf{i}^3$ into the operator reciprocal to $\mathbf{i}^1$. Eq. (15) can be interpreted geometrically; the triple product represents a three-dimensional volume, and in four dimensions this should be represented by a vector in the remaining dimension. To complete the analogy the quadruple product is a pure scalar,

$$\mathbf{i}^\alpha \times \mathbf{i}^\beta \cdot \mathbf{i}^\mu \cdot \mathbf{i}^\nu = e^{\alpha\beta\mu\gamma}\mathbf{i}_\gamma \cdot \mathbf{i}^\nu = e^{\alpha\beta\mu\nu}. \tag{17}$$

Reciprocal Sets of Four-Vectors

Consider any set of four-vectors $\mathbf{A}^\rho$, $\rho = 0, 1, 2, 3$, and require that no one of them shall be expressible in terms of components in the directions of the other three. One at least must be time like, as indicated by the zero suffix. Such a set will be called *complete*. The quadruple product of these four-vectors equals the determinant of the set of their components,

$$\mathbf{A}^0 \times \mathbf{A}^1 \cdot \mathbf{A}^2 \cdot \mathbf{A}^3 = |A^{\mu\nu}| = A, \quad \text{say}, \tag{18}$$

where

$$\mathbf{A}^\rho = A^{\mu\rho}\mathbf{i}_\mu. \tag{19}$$

The reciprocal set $\mathbf{A}_\rho$ to $\mathbf{A}^\rho$ will be obtained from

$$\mathbf{A}^\alpha \cdot \mathbf{A}_\beta = \delta_{\alpha\beta}, \tag{20}$$

giving

$$\mathbf{A}_\alpha = (1/3!A)e^{\alpha\beta\gamma\delta}\mathbf{A}^\beta \times \mathbf{A}^\gamma \cdot \mathbf{A}^\delta. \tag{21}$$

Eq. (16) is a special case of this. The reciprocality of these sets is mutual. Writing for the components of $\mathbf{A}_\rho$,

$$\mathbf{A}_\rho = A_{\mu\rho}\mathbf{i}^\mu, \tag{22}$$

and

$$A^* \equiv |A_{\mu\rho}|, \tag{23}$$

we have

$$\mathbf{A}^\alpha = (1/3!A^*)e^{\alpha\beta\gamma\delta}\mathbf{A}_\beta \times \mathbf{A}_\gamma \cdot \mathbf{A}_\delta. \tag{24}$$

The cross products also have simple forms,

$$\mathbf{A}^\alpha \times \mathbf{A}^\beta = A e^{\alpha\beta\delta\gamma}\mathbf{A}_\gamma \mathbf{A}_\delta \tag{25}$$

and

$$\mathbf{A}_\alpha \times \mathbf{A}_\beta = A^* e^{\alpha\beta\delta\gamma}\mathbf{A}^\gamma \mathbf{A}^\delta. \tag{26}$$

Eq. (13) is a special case of these. It is also worth noting that

$$A^* A = 1. \tag{27}$$

Dyadic Operators and Rotations

All familiar theorems developed by Gibbs[2] have their exact analogs in the present analysis. Some interesting examples follow. The idemfactor can be written in the alternative forms,

$$\mathbf{I} = \mathbf{A}^\alpha \mathbf{A}_\alpha = \mathbf{i}^\alpha \mathbf{i}_\alpha = \mathbf{i}_\alpha \mathbf{i}^\alpha, \tag{28}$$

$$\mathbf{I} = \mathbf{i}_0\mathbf{i}_0 - \mathbf{i}_1\mathbf{i}_1 - \mathbf{i}_2\mathbf{i}_2 - \mathbf{i}_3\mathbf{i}_3. \tag{29}$$

Any dyadic can be expressed as a sum of four dyads whose postfactors (or prefactors) can be any selected complete set of four-vectors. A dyadic will operate as a pure rotation without strain, if and only if it is the reciprocal of its conjugate, and if the determinant of its mixed components is positive unity. To illustrate this, consider the dyadic

$$\mathbf{R} = \mathbf{i}^{\alpha\prime}\mathbf{i}_\alpha, \tag{30}$$

where $\mathbf{i}^{\alpha\prime}$ are the reciprocals of another complete set of unit vectors $\mathbf{i}_\alpha'$. Note that on account of Eqs. (6) and (8) being true of $\mathbf{i}_\alpha'$ as for $\mathbf{i}_\alpha$, we shall have

$$\mathbf{R} = \mathbf{i}^{\alpha\prime}\mathbf{i}_\alpha = \mathbf{i}_\alpha'\mathbf{i}^\alpha. \tag{31}$$

The conjugate to $\mathbf{R}$ will be

$$\overline{\mathbf{R}} = \mathbf{i}_\alpha\mathbf{i}^{\alpha\prime} = \mathbf{i}^\alpha\mathbf{i}_\alpha'. \tag{32}$$

Using the second form of $\mathbf{R}$ and the first form of $\overline{\mathbf{R}}$, and remembering Eqs. (7) and (28), we shall obtain

$$\overline{\mathbf{R}} \cdot \mathbf{R} = \mathbf{i}_\alpha'\mathbf{i}^\alpha \cdot \mathbf{i}_\beta\mathbf{i}^{\beta\prime} = \mathbf{I}. \tag{33}$$

Dyadics of the form

$$\mathbf{o} = \mathbf{e}^{\alpha\prime}\mathbf{i}_\alpha, \tag{34}$$

where $\mathbf{e}^{\alpha\prime}$ are a non-unit set, will introduce strain, or curvature.

Lorentz Rotations

A dyadic in which $\mathbf{i}_0' = \mathbf{i}_0$ will be a familiar spacelike rotation, but a dyadic of the type

$$\Lambda = \mathbf{i}_0'\mathbf{i}^0 + \mathbf{i}_1'\mathbf{i}^1 + \mathbf{i}_2\mathbf{i}^2 + \mathbf{i}_3\mathbf{i}^3 \tag{35}$$

represents a rotation in the plane of x^1 and t. It

[2] Gibbs-Wilson, *Vector analysis*, Chaps. 5 and 6.

has the effect of rotating all vectors upon which it operates in the same way as $\mathbf{i}_0$, $\mathbf{i}_1$ would have to be rotated so as to coincide with $\mathbf{i}_0'$, $\mathbf{i}_1'$, respectively:

$$\mathbf{\Lambda}\cdot\mathbf{i}_1=\mathbf{i}_1'; \quad \mathbf{\Lambda}\cdot\mathbf{i}_0=\mathbf{i}_0'. \tag{36}$$

Because of the completeness required of the new set $\mathbf{i}_0'$, $\mathbf{i}_1'$, $\mathbf{i}_2$, $\mathbf{i}_3$, we must be able to write

$$\begin{aligned}\mathbf{i}_0'&=a\mathbf{i}_0+b\mathbf{i}_1,\\ \mathbf{i}_1'&=c\mathbf{i}_0+d\mathbf{i}_1.\end{aligned} \tag{37}$$

Eqs. (2), which are true for $\mathbf{i}_0'$, $\mathbf{i}_1'$, $\mathbf{i}_0$ and $\mathbf{i}_1$, give

$$a^2-b^2=1, \quad d^2-c^2=1, \quad ac-bd=0.$$

Therefore, there exists an angle ϕ such that

$$\left.\begin{aligned}&\mathbf{i}_0'=\cosh\phi\mathbf{i}_0+\sinh\phi\mathbf{i}_1,\\ \text{and either}&\\ &\mathbf{i}_1'=\sinh\phi\mathbf{i}_0+\cosh\phi\mathbf{i}_1\\ \text{or}&\\ -&\mathbf{i}_1'=\sinh\phi\mathbf{i}_0+\cosh\phi\mathbf{i}_1.\end{aligned}\right\} \tag{38}$$

The second alternative will be ignored because $\phi\rightarrow 0$ when $\mathbf{i}_1'\rightarrow-\mathbf{i}_1$, and therefore corresponds to a reflection rather than a rotation. The determinant of the dyadic would be negative unity. The components of $\mathbf{\Lambda}$ are thus given by

$$\mathbf{\Lambda}=\lambda_\nu{}^\mu\mathbf{i}_\mu\mathbf{i}^\nu, \tag{39}$$

with

$$\lambda_\nu{}^\mu=\begin{array}{c|cccc} & \mu\rightarrow & & & \\ \nu\downarrow & \cosh\phi & \sinh\phi & 0 & 0\\ & \sinh\phi & \cosh\phi & 0 & 0\\ & 0 & 0 & 1 & 0\\ & 0 & 0 & 0 & 1\end{array} \tag{40}$$

and the determinant $\Lambda\equiv|\lambda_\nu{}^\mu|=1$.

Consider any given base system $\mathbf{i}_\alpha$, and let it be rotated by means of a dyadic operator, such as is given by Eq. (40) or (35), into a new set. Any given interval $d\mathbf{s}$, not operated upon by $\mathbf{\Lambda}$, will remain fixed and can be expressed in terms of components with respect to either system of units. These components will thus undergo the Lorentz transformation as we pass from the reference set $\mathbf{i}_\alpha$ to the rotated set $\mathbf{i}_\alpha'$. Thus

$$\begin{aligned}d\mathbf{s}&=dx^\alpha\mathbf{i}_\alpha=dx^{\alpha\prime}\mathbf{i}_\alpha'=dx^{\alpha\prime}\mathbf{\Lambda}\cdot\mathbf{i}_\alpha\\ &=dx^{\alpha\prime}\lambda_\nu{}^\mu\mathbf{i}_\mu\mathbf{i}^\nu\cdot\mathbf{i}_\alpha=dx^{\alpha\prime}\lambda_\alpha{}^\mu\mathbf{i}_\mu,\end{aligned}$$

so that

$$dx^\mu=\lambda_\alpha{}^\mu dx^{\alpha\prime}, \tag{41}$$

which, with Eqs. (39) and (40), gives the familiar equations of the Lorentz transformations in special relativity.[3]

The rotation $\mathbf{\Lambda}$ can be put in a form explicitly containing only the two vectors which suffer change. Remembering Eq. (30), and using Eq. (38) in (35), we obtain

$$\begin{aligned}\mathbf{\Lambda}=\mathbf{I}+(\cosh\phi-1)(\mathbf{i}_0\mathbf{i}^0+\mathbf{i}_1\mathbf{i}^1)\\ +\sinh\phi(\mathbf{i}_0\mathbf{i}^1+\mathbf{i}_1\mathbf{i}^0).\end{aligned} \tag{42}$$

Making use of Eqs. (6) and (8), and writing $\mathbf{a}$ in place of $\mathbf{i}_1$ to indicate that any unit vector may be employed which can be put in the form

$$\mathbf{a}=l\mathbf{i}_1+m\mathbf{i}_2+n\mathbf{i}_3, \tag{43}$$

we derive

$$\begin{aligned}\mathbf{\Lambda}=\mathbf{I}+(\cosh\phi-1)(\mathbf{i}_0\mathbf{i}_0-\mathbf{aa})\\ -\sinh\phi(\mathbf{i}_0\mathbf{a}-\mathbf{a}\mathbf{i}_0).\end{aligned} \tag{44}$$

The general Lorentz transformation for relative velocity in any direction $\mathbf{a}$ is immediately derivable from this expression.[4]

Conclusions

By adopting a more rational system of base units, it proves possible to make use of all the advantages of both tensor and vector analysis in the treatment of special relativity theory, without having to introduce the unsatisfactory notion of imaginary time required on the scheme of Minkowski.

The Lorentz transformation is regarded as the consequence of a dyadic rotation of the reference system of base units, and is expressed by an obvious generalization of the rotation dyadic in three dimensions.

None of the results are essentially new, but the method of approach has proved valuable in the presentation of the subject, and it also leads to an attractive notation for use in the general theory of relativity which we hope to report later.

[3] Tolman, *Relativity, thermodynamics and cosmology*, Sec. 8.
[4] Cunningham, *The principle of relativity.*

Truth doesn't depend upon the number of its believers.—Zsolt de Harsanyi, *The Star Gazer.*

A Convenient Vibration Source of Variable Frequency for Melde's Experiment

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MELDE'S experiment, as it is usually performed, is subject to decided limitations; the source of vibrations frequently used, is an electrically driven tuning fork which permits a quantitative study to be made of the relation between wave-length and the stretching force on the string, or wave-length and mass per unit length, for one frequency only; namely, that of the fork. Usually the frequency of the fork is not measured directly, since this would involve a considerable amount of auxiliary apparatus, but rather the value is taken as marked on the fork. The frequency cannot be varied, unless several forks are used, and it is not common to have an assortment of electrically driven forks on hand for elementary experiments. Furthermore, electrically driven tuning forks too often behave temperamentally and, in the middle of a laboratory period, suddenly refuse to operate, due to burned contacts or some other less obvious but equally inconvenient cause.

The scope of Melde's experiment can be much extended, both for demonstration purposes and for quantitative experiments, by using a properly designed eccentric on a variable speed rotator as a source of vibration. Various devices for attachment on rotators have been used but they seem to be excessively cumbersome and quite expensive. Some years ago the senior author conceived the idea of obtaining the desired results with the device shown in Fig. 1. The disk A was one of the pulleys that goes with the standard Cenco rotator. A metal pin B with a head, such as a wire nail, was run through an eccentric hole at C and bent into the form of a hook. The string S to be vibrated was then secured, running out in a horizontal direction. Rotation of the motor carried the end of the vibrating string through a circular motion. In order, however, to avoid the twisting of the string which ordinarily would occur with such a device, the hook member B was prevented from rotating about its own axis by a rubber band R extending to some fixed point, such as the base of the rotator. Thus the string had only a circular motion and no twisting. The wave in the string was the equivalent of a circularly polarized wave. This does not in any way alter the theory of Melde's experiment, and for demonstration purposes has the distinct advantage that the appearance of the standing wave is equally good from all planes containing the string. If it is desired to have a plane-polarized wave, this can be obtained by placing a slotted barrier in front of the rotating disk.

With this arrangement it is possible to vary the frequency continuously over the full range of the rotator, and to measure it directly by means of a revolution counter. Thus it is possible by direct measurement to study all the relations

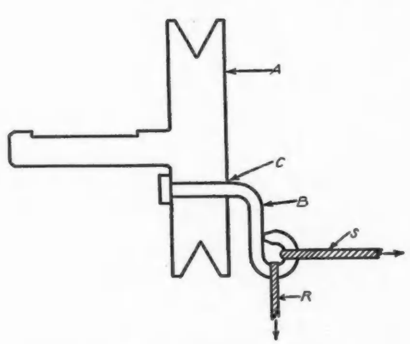


FIG. 1. Simple vibration source of variable frequency.

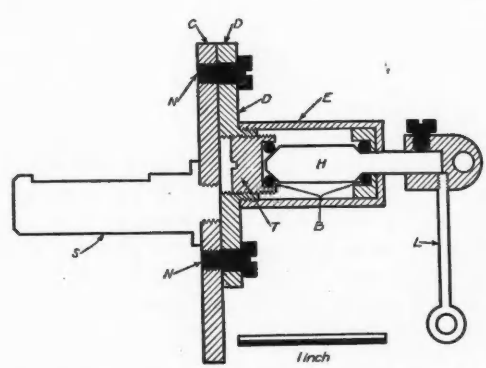
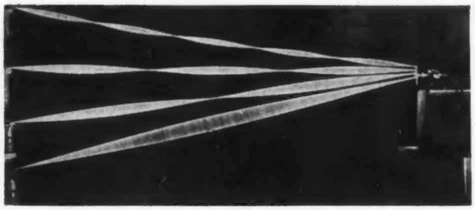


FIG. 2. A more elaborate variable-frequency source.



(a)

(b)

FIG. 3. Photographs of typical demonstrations using the eccentric.

between frequency, force, wave-length and mass of a stretched string.

While the device is exceedingly effective and of nearly zero cost, it develops that, if used in laboratory work where a large number of students must be served in the course of the year, the pin B or the hole C will rather quickly wear by friction. Consequently, a more elaborate arrangement was designed and has proved to be highly satisfactory. The main characteristics are indicated in Fig. 2.[1]

The shaft S is made to fit into the Cenco rotator head and carries a rectangular plate C about 2 in. long. To this is adjustably secured, by screws N, a plate D and cylinder E in which standard ball-bearing races BB are mounted and in which the hardened steel shaft H turns. Any play which may develop in H may be taken up by a set screw T, on which there is a lock nut not shown in the diagram. The arm L serves to keep the shaft H from rotating when a string that twists easily is being used. A small weight hung from the end of L, or a rubber band attached from it to a point on the table, accomplishes this purpose. The amount of eccentricity of the shaft H which we have found satisfactory is $\frac{1}{4}$ in., though this may easily be made adjustable by cutting slots for the screws N and changing the position of D with respect to C. The dimensions given resulted largely from chance estimate; and, while our eccentrics seem to be standing up well under hard use, it may be desirable in the future to employ somewhat larger ball-bearing races.

Here again it will be noted that the vibrations set up by the eccentric are *circular* rather than *linear*, and consequently the wave in the string is circularly polarized. In Fig. 3(a) the four cords, operated in parallel, are of the same mass per unit length and the stretching forces are adjusted to give the different number of segments. In the photograph the nodes do not appear very sharp. This is due to irregular shifting of the nodal points, during the two-minute exposure, because of varying frictional forces at the pulley. Actually the nodal points, at any one time, are sharp. Fig. 3(b) illustrates a situation which we have found very instructive and apparently not mentioned elsewhere. The loops are of unequal length because the string in this case consists of two sections of quite different masses per unit length. It shows strikingly the change in velocity of the wave as one passes at the point D from one medium to another of different "density."

The type of string used for demonstration is

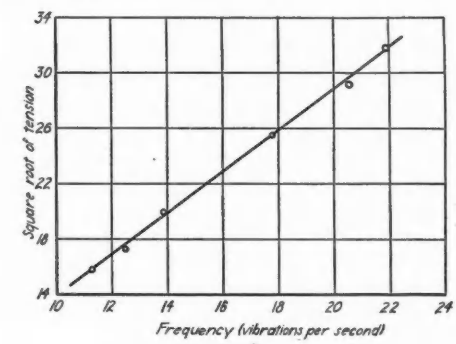


FIG. 4.

TABLE I. *Typical data.**

No. of Segments	No. of Revolutions	Time (sec)	Frequency (vib/sec) Measured	Frequency Computed
1	200	56	3.57	3.63
2	500	68	7.35	7.25
3	800	72	11.1	10.9
4	1000	68	14.7	14.5
5	1000	54	18.5	18.4
6	1600	63.5	25.2	25.2

* Length of string, 374 cm; stretching force, 1000 gm.

[1] The details of construction are largely due to D. W. Mann, of Lincoln, Massachusetts.

of little consequence, as long as it is light enough in color and heavy enough in texture for good visibility. For performing a quantitative experiment, where it is desirable to have a string of sufficient mass to avoid unreasonably small stretching forces at the relatively low frequencies obtainable from the rotator, we have found lightweight sash cord or rubber and cloth insulated copper wire (similar to lamp cord wire) very satisfactory. Table I gives typical sets of data taken with only moderate care, using the insulated copper wire for a string. The results were obtained while keeping the length and stretching force constant, but varying the frequency. Fig. 4 shows the experimental relation obtained between stretching force and frequency, the number of loops (therefore the wave-length) being kept constant.

A Simply Constructed Source of Ultraviolet Continuum

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AN important part of the physics major's undergraduate curriculum should be individual laboratory work in what might be called "sub-research." Here he works with technics which are usually quite well standardized, but which should give him valuable experimental training and introduce him to the type of thinking and action that he will use in his bona fide research activities in graduate student days. A timely subject, and one that is well suited to such semi-independent experimental work, is the investigation of the selective absorption spectra of solutions. This has the added advantage that it can be carried on in a laboratory with rather limited facilities.

The necessary equipment consists of a small quartz spectrograph, absorption cells and a source of continuous spectrum. Absorption cells can be constructed readily from glass or brass tubing, with waxed-on windows of glass or quartz. An inexpensive and satisfactory source of continuum for the visible region of the spectrum is a low voltage, high current, tungsten-filament lamp such as the 10-v, 75-w lamp used to illuminate the sound track in moving picture projectors. A good source of continuum for the ultraviolet region is not secured so easily, however. The most satisfactory one seems to be the hydrogen discharge tube. Hydrogen tubes constructed entirely of fused quartz are on the market, but their price may be too high for a modest departmental budget. Various "home-made" tubes are described in the literature,[1] but all are of quite complicated design, requiring quartz capillaries, ring seals, or cumbersome water-cooling devices. We have designed a simple, easily constructed, all-Pyrex tube that requires no difficult seals and is light, rugged and easily transportable. In addition to the uses to which such a tube may be put, its construction is simple enough to be suitable for a student glass-blowing project.

Figure 1 is a diagram of the tube, which is essentially a large-size, conventional end-on discharge tube. The tube proper consists of two electrode compartments of 3 cm diameter joined by a 6-mm capillary which is approximately 15 cm in length. On one end of the tube is waxed a fused quartz window; on the other end is sealed a large bulb to increase the volume of the tube, thereby steadying the discharge and retarding

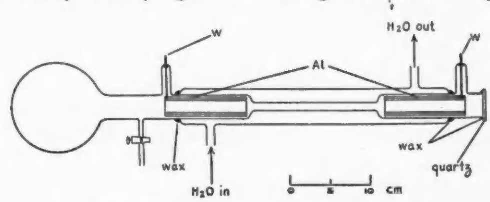

FIG. 1. Water-cooled hydrogen discharge tube, made entirely of Pyrex glass except for the fused quartz window, the rolled sheet aluminum electrodes, the tungsten lead-in wires and the waxed joints.

* Now at The Ohio State University.

[1] N. D. Smith, J. Opt. Soc. Am. 28, 40–45 (1938) Fifteen references are given.

hardening. The electrodes consist of rectangular sheets of thin aluminum which are rolled spirally to fit tightly against the glass. The tungsten leads, which make mechanical connection with the electrodes, are sealed in through small side-arms. A stopcock is added on a small side tube.

All of the capillary and as much of the electrode compartments as possible are covered by the water jacket, which is a Pyrex tube of 4 to 5 cm diameter. This tube is drawn down at the ends to the size of the electrode compartments and is waxed to them with Picein cement. The various dimensions indicated in Fig. 1 and quoted above are all arbitrary, within reasonable limits.

The sequence of steps in the construction of the tube is as follows: (1) the electrode compartments and capillary are joined, and the electrodes are slipped inside their compartments; (2) the tungsten lead-in wire at the rear of the tube (the large bulb end) is added, after which the bulb and stopcock are affixed; (3) the water jacket is slipped into place and waxed to the electrode compartments, and the front tungsten lead-in wire is installed; (4) the quartz window is waxed onto the front end of the tube, which has previously been ground flat on a glass plate, using carborundum powder.

To prepare the tube for operation it is exhausted as completely as possible, then is flushed several times with nitrogen-free hydrogen and, when pumped down for the last time, is left with enough hydrogen to be just below striking pressure. Under normal circumstances it will run many hours before requiring refilling.

A satisfactory power source is a commercial pole-type distribution transformer rated at 1.5 kva, 230–4000 v. A resistance of approximately 5000 ohms must be put into the secondary circuit, in series with the tube, in order to hold the tube current to 300–500 ma and avoid overloading the transformer. At this current the tube will not overheat, and will produce a continuum strong enough for low dispersion spectrographs.

Tubes of this type have been used for several months in photographing the absorption band of vitamin A at 3280A, and have performed satisfactorily in every respect.[2]

[2] Nelson and Ballard, Phys. Rev. **57**, 253A (1940).

The Dependence of Thermodynamic Functions on the Mass of the System

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In thermodynamic theory the variables which enter are commonly divided into two classes, *intensive* and *extensive*. The distinction is made according to whether the variable concerned is independent of the mass of substance present, as the pressure P and the temperature T, or whether it is a function of the mass, as the energy E and the entropy S. The former are the intensive and the latter the extensive variables.

It is generally assumed to be a self-evident fact that, for given values of the intensive variables, the extensive variables are proportional to the amount of substance present. This means, for example, that if we express the extensive variable V as a function of P, T and N, the number of mols of substance present, we have a functional relation of the form

$$V = N\cdot f(T, P)$$

or, solving for P,

$$P = \phi(T, N/V) \tag{1}$$

as the equation of state. In words, it means that N and V enter only in the combination N/V, and not separately. Similarly, for the total energy,

$$E = N\cdot \xi(P, T), \tag{2}$$

with equations of analogous type for the other thermodynamic functions.

This hypothesis has such an immediate appeal and such an evident experimental justification that we let it pass virtually unquestioned into thermodynamic theory. It is sometimes even implied that one can prove its validity from pure thermodynamic theory alone, which is not the case. Although it is consistent with the laws of

thermodynamics, it is not necessary for their validity.

In order to arrive at an understanding of the nature of the problem, it is necessary to adopt the point of view of statistical mechanics. We shall study only the case of an ideal gas of structureless mass particles, and shall show that in principle the hypothesis is not correct even for this simple system. However, on calculation, we find that the deviations from it are of the nature of quantum effects, and are negligible except possibly under unusual circumstances.

Since we need not derive the statistical-mechanical formulas here, it will be convenient to state them in a form suitable for all statistical distributions simultaneously. If the energy levels of a particle in the container are designated as $\epsilon_1, \epsilon_2, \cdots$ and the number of different states associated with the rth energy level by $\omega(\epsilon_r)$ (degree of degeneracy), then the expressions for the total number of particles and the total energy of the gas are

$$n=\lambda\sum_r \frac{\omega(\epsilon_r)e^{-\epsilon_r/kT}}{1+a\lambda e^{-\epsilon_r/kT}},\qquad E=\lambda\sum_r \frac{\omega(\epsilon_r)\cdot\epsilon_r\cdot e^{-\epsilon_r/kT}}{1+a\lambda e^{-\epsilon_r/kT}}, \tag{3}$$

$a=+1$, Fermi-Dirac statistics
0, Maxwell-Boltzmann statistics
-1, Bose-Einstein statistics.

The quantity λ is to be found from the first of Eqs. (3) and then substituted in the second.

In order to satisfy the conditions of the hypothesis it is necessary that $\omega(\epsilon_r)$ be proportional to V, the volume of the container holding the gas; for this will make λ a function of T and n/V, while E will be of the form

$$E=V\times\text{a function of } T \text{ and } n/V.$$

The equation of state is given by the general relation[1]

$$P=2E/3V,$$

from which we get a relation of the form of Eq. (1).

[1] The generality of this relation for an ideal gas is restricted only to the extent that the stress acting on the surface may differ from a true uniform hydrostatic pressure. For a cubical container it is strictly accurate for all magnitudes of the volume and for all statistics.

Now the possible energy levels of a particle in a box are easily found from the Schrödinger equation to be given by the expression[2]

$$\epsilon(m_1, m_2, m_3) = \frac{h^2}{8M}\left[\left(\frac{m_1}{\alpha}\right)^2+\left(\frac{m_2}{\beta}\right)^2+\left(\frac{m_3}{\gamma}\right)^2\right], \tag{4}$$

where we have assumed the container to have the form of a rectangular parallelepiped of sides α, β, γ and the mass of a particle to be M. The quantum numbers m_1, m_2, m_3 assume the values $1, 2, 3\cdots$. The degree of degeneracy $\omega(\epsilon)$ will be the number of choices of the m's which can be made to fit this equation for a given value of ϵ.

It is easy to show that, for large volumes $V(=\alpha\beta\gamma)$ of the container, the energy differences between the separate levels are very small, and the number of levels having energies in the interval between ϵ and $\epsilon+\Delta\epsilon$ is approximately

$$g_0(\epsilon)\Delta\epsilon=(2\pi/h^3)V(2M)^{3/2}\epsilon^{1/2}\Delta\epsilon. \tag{5}$$

To this degree of approximation we can replace the sums in Eqs. (3) by integrals, and observe at once that the conditions implied in the theorems are satisfied. This is the approximation universally employed in kinetic theory, and is the one implied in the condition that a volume of magnitude h^3 in the phase space of a particle is equivalent to one quantum state. To this degree of approximation, then, the statistical-mechanical approach justifies the hypothesis made in thermodynamic theory.

It is interesting to observe that this argument made no use of the form of statistics involved, and consequently would apply to a Fermi-Dirac or a Bose-Einstein gas as well as to one described by the Maxwell-Boltzmann statistics. At first sight this seems curious, since the first two forms of statistics imply special symmetry characteristics between all of the molecules of the gas. For example, suppose we have two equal volumes of the same kind of gas having the same density and temperature, but with a partition separating them, so that the molecules in the two parts are to be considered as quite independent of one another. If we apply Eqs. (3)

[2] This problem is discussed in almost all books on quantum mechanics.

and (5) to the two parts separately, it is clear at once that no change in pressure or energy will occur if the partition be removed and the two gases be allowed to fill the whole container.

On the other hand, since Eq. (5) is an approximate formula, valid only asymptotically for large volumes, it is of interest to make an estimate of the error involved in its use. It has been remarked many times that the problem of finding $\omega(\epsilon)$ from Eq. (4) is identical with that of finding the distribution of standing acoustic waves in a room having the same shape as the container of gas. We need only associate the momentum p of a particle with a wave of length

$$\Lambda = h/p \quad \text{(de Broglie wave-length).}$$

This latter problem has been discussed recently in the acoustic literature, from which we can borrow the result. It will suffice for our purposes to consider a cubical container, for which the number of energy levels lying between ϵ and $\epsilon+\Delta\epsilon$ is given in third approximation by[3]

$$g(\epsilon)\Delta\epsilon = g_0(\epsilon)\cdot\left\{1+\frac{h}{8V^{1/3}(2M\epsilon)^{1/2}}+\frac{h^2}{8\pi V^{2/3}2M\epsilon}\right\}\Delta\epsilon.$$

It is clear from the form of this expression that it will affect only the distribution of the low-lying energy levels where the denominators of the correction terms become appreciable. Of course, it must be remembered that, from Eq. (5), $g_0(\epsilon)$ varies as $\epsilon^{1/2}$.

If we insert this expression in Eqs. (3), and use the Maxwell-Boltzmann statistics ($a=0$), we can carry out the results explicitly and find

$$n = \lambda\int_0^\infty g(\epsilon)\cdot e^{-\epsilon/kT}d\epsilon = \left(\frac{\lambda V}{\eta^3}\right)\left\{1+\frac{\eta}{4V^{1/3}}+\frac{\eta^2}{4V^{2/3}}\right\},$$

$$E = \lambda\int_0^\infty \epsilon g(\epsilon)e^{-\epsilon/kT}d\epsilon$$

$$= \tfrac{3}{2}kT\left(\frac{\lambda V}{\eta^3}\right)\left\{1+\frac{\eta}{6V^{1/3}}+\frac{\eta^2}{12V^{2/3}}\right\},$$

$$P = \frac{2E}{3V} \cong \left(\frac{nkT}{V}\right)\left\{1-\frac{\eta}{12V^{1/3}}+\cdots\right\},$$

$$\eta = h/(2\pi MkT)^{1/2}.$$

The second and third terms in the brackets are the correction terms. In order to evaluate them, we observe that η has the dimensions of a length; in fact, it is the de Broglie wave-length of a particle having an energy πkT. Using for M the mass of a hydrogen molecule, we find

$$\eta \approx 1.2T^{-1/2}\times 10^{-7}\ \text{cm}.$$

This shows that the ratio $\eta/V^{1/3}$ will be small so long as the effective volume allowed to a molecule is large compared with molecular volumes themselves, and T is not too small.

In the case of highly degenerate gases subject to the Bose-Einstein statistics, the effect might be of at least theoretical importance, since a large proportion of the particles would be concentrated in the lowest energy levels. In actual gases, however, condensation occurs before deviations from the Maxwell-Boltzmann statistics become appreciable.

A little consideration will show that, for a gas subject to the Fermi statistics, the correction would be even smaller, since a good fraction of the particles would be in the higher energy states; and we have already remarked that the correction is appreciable only for the distribution of the relatively few low energy levels.

In the case of a homogeneous solid phase, a calculation can be made, using the Born-Debye theory of lattice oscillations, along similar lines and with a similar conclusion.

Consequently, it seems that, for homogeneous phases where unique equations of state are the rule, the assumption underlying the thermodynamic treatment should be valid to a high degree of approximation. In particular cases more careful consideration might be required.[4]

[3] R. H. Bolt, J. Acous. Soc. Am. **10**, 228 (1939); Dah-You Maa, J. Acous. Soc. Am. **10**, 235 (1939). The formula used here is adapted from Eq. (2) of the latter reference.

[4] *Note added in proof.*—In a paper just received, K. Husimi, Proc. Phys.-Math. Soc. Japan **21**, 759 (1939), the distribution of energy levels is rediscussed, and considerations similar to the above are given for a Bose-Einstein gas.

Dans les champs de l'observation le hazard ne favorise que les esprits préparés.—PASTEUR.

Student Projects in Physics at Kalamazoo College

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Even among supposedly homogeneous groups of students the minor regimentation which must be imposed in order to cover specific course content leads to difficulties due to disparity in ability. Partly to adjust this inequality, but largely to encourage more of the zest and pioneering spirit of research in undergraduates, we offer at this college a course in individualized special work in physics. This course is in keeping with the general college policy of emphasizing needs of the individual student, and aims to augment the solid foundation of formal theory and prescribed laboratory work with experience in facing problems in which all the difficulties have *not* been anticipated.

The student is allowed to choose, out of as large a selection as he and his adviser can assemble, that project in which he takes most interest. The adviser endeavors to use discretion in restraining the student from making a choice which is beyond his capacity, and thus, to some extent, "tempers the wind to the shorn lamb." The problem usually consists of the construction of some major piece of apparatus or the performance of a group of related experiments in some unfamiliar branch of physics, such as vacuum tubes, photography or acoustics. Occasionally a student may wish to study only the theory of some new field. As far as possible, the student is left to his own resources. Naturally, students of higher caliber go farther, but even those who are somewhat apathetic to formal class work in physics show much increased interest and comprehension when working on subjects of their own choice. As a useful by-product, students frequently acquire skill in shop technics, for which a liberal arts curriculum may provide no other opportunity. Most of them are obliged to refer to the literature in the course of planning their work and must eventually learn to thread their way through the intricacies of the subject index of *Science Abstracts*.

It was felt that it might be of interest to other teachers to see the fruits of one year's effort (1938–1939) by students in the special problems course. The following projects were completed:

1. Paschen-type grating spectrograph.
2. Prism spectrograph.
3. Sound diffraction demonstration set.
4. Geiger-Müller counter.
5. Demonstration cutaway automobile transmission.
6. Physics shop.

In addition to these an experiment was completed in which a 13-m Foucault pendulum was used to evaluate g with higher precision than by the typical elementary laboratory methods and to determine irregularities of the local telechron (controlled power frequency) signals. There was worked out also a new type of miniature Kundt tube.[1] Approaching completion at present are a student-built dark room and an air-driven ultracentrifuge.[2] The probable schedule of projects for the current year includes the color photography of spectra on the previously mentioned Paschen spectrograph, high speed photography[3] and the construction of an "artificial larynx."[4] The first six projects will be briefly described.

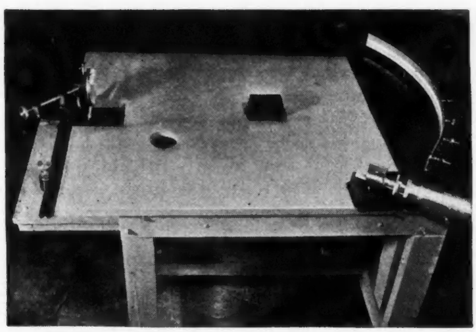

Fig. 1. Paschen-type mounting for a concave grating spectrograph.

[1] H. Seifert, Am. J. Phys. (Am. Phys. T.) **7**, 421 (1939). This was not a student project but was worked out in collaboration with students.
[2] Of the type described by J. W. Beams in Rev. Mod. Phys. **10**, 247 (1938).
[3] A. B. White, Electronics, **10**, 18 (1937).
[4] A controlled relaxation oscillator feeding sound energy into a mouthpiece.

FIG. 2. Sound diffraction demonstration set.

The Paschen-type mounting for a concave grating (Fig. 1), built by Mr. George Baldwin, consists of a heavy reinforced wooden table on which is mounted a tripod grating holder permitting rotation around three mutually perpendicular axes. The grating is a 25×35-mm Wallace replica. The film holder, adapted to 35 mm film, is backed by a flexible steel strip which can be permanently adjusted to fit the focal curve of the grating. A precision slit is mounted on the end of a light-tight tube leading to the source in an adjacent room. The black object in the center of the table is the pivot support for a rotating arm used to bring the various components into approximate position on the Rowland circle. The results obtained by Mr. Baldwin with this spectrograph include clean-cut spectra of Fe, He and Ne extending from about 3800A to 7300A on strips of film about 60 cm long, as well as some interesting color photographs of Fe and He spectra on Dufay film.

The prism spectrograph was built by Mr. Adolphus Perry. The camera is an Eastman view camera on demountable supports so that it may be removed and used elsewhere. The prism table is an old one. A special mounting was built for the low speed, long focus camera lens. Spectra taken on this camera are used by students as guides for visual identification of spectral lines in the first-year physics laboratory.

In Fig. 2 is shown the major portion of a sound diffraction demonstration outfit[5] built by Mr. William Weber and Mr. Wilfred Shale. To the left is the source, set up as a double slit, as well as the amplifier and oscillograph on which the interference pattern is observed. On the right is a "half-silvered mirror" (window screen) and total reflector. A Lloyd's mirror type of reflector can be seen in the background. The pick-up microphone in the foreground is mounted on an optical bench. The builders of this outfit have performed most of the related demonstrations on acoustics described in Sutton's book.[6] It is planned to use it for quantitative experiments in connection with a course in optics.

The Geiger-Müller photon counter was built by Mr. James Barclay. The high voltage supply, metal counter tube and amplifier-trigger circuit were built as separate units. Mr. Barclay also made several glass counter tubes according to a special method,[7] and rewound a low impedance Cenco counter to make it sufficiently high in impedance to operate in the thyratron tube output circuit. It was possible with this counter to observe the increase in counting rate produced by the approach of a radium-dial clock simultaneously by means of numerical counter, loudspeaker, and oscillograph. It was not found possible to make the circuit work with the constants given by Neher.[8] These apparently had to be adjusted considerably to fit local circumstances.

The cutaway automobile transmission[9] was mounted by Mr. Charles Thompson. A modern transmission was donated to the department by the Studebaker Corporation. Mr. Thompson removed a portion of the casing and the overdrive unit, and then mounted the assembly in convenient form for elementary laboratory use. The gears are shifted by small levers, part of the steering column shift. This apparatus is now used in a regular elementary laboratory experiment.[10]

The new shop, which was financed cooperatively by the departments of physics and chemistry, was put in systematic and usable condition mainly through the efforts of Mr. Barclay. However, it represents a "project" of the entire science group, and upon its adequacy the success of other individual projects largely depends.

Without the willing financial assistance of the college administrators, as well as the generous help and continued encouragement of my senior colleague, Professor J. W. Hornbeck, these various projects could not have been realized. To these men, in behalf of the enthusiastic group of students of whom he writes, the author expresses his gratitude.

[5] Patterned closely after the design of Schilling, Am. J. Phys. (Am. Phys. T.) 4, 27 (1936).

[6] Sutton, *Demonstration experiments in physics* (McGraw-Hill, 1938).

[7] Strong, *Procedures in experimental physics* (Prentice-Hall, 1938), p. 270.

[8] Ref. 7, pp. 277, 283.

[9] Blackwood and Hutchisson, Am. J. Phys. (Am. Phys. T.) 1, 41 (1933).

[10] Millikan, Roller and Watson, *Mechanics, molecular physics, heat and sound* (Ginn, 1937), p. 83.

It is the great beauty of our science, that advancement in it, whether in a degree great or small, instead of exhausting the subject of research, opens the doors to further and more abundant knowledge, overflowing with beauty and utility.—FARADAY.

Measuring the Results of Instruction in College Physics

A Summary Report on the National

College Physics Testing Program, 1933–1939

THE National College Physics Testing Program, the first large-scale cooperative departmental measurement project to be undertaken in American colleges, has been made possible through the joint efforts of the committee on tests of the American Association of Physics Teachers and the Cooperative Test Service of the American Council on Education. It was inaugurated with the publication of a series of pre-study tests in physics[1] in the fall of 1933. A comparable series of post-study tests was issued in the spring of 1934. Similarly, from 1934–35 through 1937–38, two new series of tests were made available each year. In 1938 the committee on tests undertook a thorough revision of the test materials, and the first examinations of the revised series (Forms A and B) were issued in 1938–39. These new forms were constructed entirely of items of known difficulties adapted from previous editions, making possible the issuance of norms *in advance of the program.* Instructors were thus enabled to interpret test results as soon as the papers were scored.

The present report reviews both the earlier and the revised series tests, and has been prepared to give physics teachers some idea of the services of the testing program, of the background and meaning of the test results, and of the effects which the program has had upon physics instruction in general. The report is based mainly upon results made available by test users to the Cooperative Test Service. In 1939, in an effort to obtain even more direct information regarding the use of tests, a special questionary was sent to a considerable number of physics teachers who had used the tests, and a tabulation of their replies provided important additional data for appraising the project.

Nature of the tests.—The examinations employed in the physics testing program have been carefully constructed. Each of the test items was studied in detail and its relative validity determined. The items, as well as the outline of the course materials which the test covers, were prepared by physics instructors to measure understanding of specific content in the elementary college physics course. Subsequent statistical analysis regarding the value of each item as a discriminator between good and poor physics students was undertaken to complete the validation studies.

The physics examinations differ from those used in other departmental measurement projects in several important particulars. For example, two separate tests are provided for each of the major subdivisions of the course. This arrangement permits a teacher to administer the tests on each subdivision immediately before the beginning of instruction and after its close. Each teacher can also select those subdivisions that best fit his particular course. The physics program is the only measurement program for which separate tests are issued covering special subdivisions, and the only one for which pre-study and post-study tests are available in a number of comparable editions.

The length of the tests has remained approximately the same from year to year. The median number of items and the median time for each unit, pre-test and post-test, for several years are as follows: mechanics, 53 items, 60 min; heat, 30 items, 30 min; sound, 16 items, 20 min; light, 35 items, 40 min; electricity, 44 items, 50 min; modern physics, 19 items, 25 min.

In the 1938 series, separate tests were issued in the mechanics of solids and the mechanics of fluids. The first of these had 32 items and required 35 min; the second had 22 items and required 25 min. The two tests together contained, therefore, approximately the same number of items as had the previous editions of the mechanics test. When the revised series tests appeared, the mechanics test was issued as a single unit again, with the number of items reduced to 50 and the time limit shortened to 50 min to provide for more convenient administration.

Validities and difficulties.—Statistical study has shown that the validities and difficulties of the individual test items have been approximately uniform from year to year. The difficulties of the *total* tests have remained relatively constant from one year to another. The student whose score falls at the 50th percentile of the distribution answers correctly from 30 to 40 percent of the total number of items. The tests thus appear "hard," but it is obviously desirable for tests such as these to have more "top" than "bottom," since the distribution of ability seems to be skewed toward the bottom. Selective factors eliminate large numbers of students at the lower levels of ability before they arrive in college to take the tests, while students in the top brackets range all the way up to the highest levels of human ability.

[1] Cooperative Physics Tests for College Students, Cooperative Test Service, 15 Amsterdam Avenue, New York, New York.

Participation in the programs.—The response of colleges to the call for participation in the testing project has been gratifying to the committee on tests. The following number of colleges have taken part each year: 1933–34, *355*; 1934–35, *269*; 1935–36, *227*; 1936–37, *254*; 1937–38, *252*; 1938–39, *270*. The continued support which the program has received from physicists over a period of years indicates that the tests are undoubtedly filling a real need. In spite of certain recognized limitations, they provide a type of information not adequately supplied by other measures of student achievement. When used in conjunction with the other and more usual methods of appraising student performance, they increase appreciably the scope and accuracy of the final evaluation.

Annual reports.—From 1934 through 1937, reports of the testing programs were published annually.[2] These reports usually included general percentile norms for all men students, abbreviated norms for women students, technical discussions of the difficulties and validities of the individual test items, distributions of college averages, studies of gains in achievement from the beginning of the course in physics to its end, various comparisons between special groups and studies of the effects of conditioning factors upon achievement in elementary physics.

The Test Results

Certain recurrent findings are discussed in the present report, and additional data from the 1938 and 1939 programs are reviewed.

National norms.—The most obviously valuable materials in the annual reports have probably been the *national percentiles*. The uses which have been mentioned for these percentiles include the following:

(1) Checking the difficulties of the tests to see that they differentiate at both the upper and lower portions of the distribution, without clustering of scores at either end.

(2) Identifying gifted students at an early stage of the course, so that they can be relieved of certain fundamental requirements, encouraged to undertake individual projects in the laboratory or in the library, or permitted to enter advanced courses immediately. In this last connection, it should be noted that some students at the beginning of instruction make scores appreciably above the class average to be expected at the end of instruction. This provides an impressive demonstration of the fact that learning is an individual process, and that conditioning factors in the background of the student are extremely important. By modifying instruction according to the pre-study test results, a teacher can go beyond the point of *talking* about adaptation of instruction to the needs and interests of each student; he has information that allows him to *do* it.

(3) Identifying students who do not possess adequate background or ability for the study of college physics, so that, by special attention, they may be saved from failure, or, if this appears impossible, encouraged to work in other fields where their efforts are more likely to be rewarded with success.

(4) Reporting the ratings of students in terms of a *national norm*. This information is particularly important to scholarship committees, graduate schools, professional employers and other institutions to which the student may desire to transfer.

(5) Placing transfer students from other institutions. When the student does not bring comparable achievement records with him, the tests may be given by the institution to which he is transferring. The distributions of college averages show such wide variability among students from physics departments in different institutions that it is highly desirable for departmental committees to possess comparable test scores as well as transcripts of grades in allocating students to sections or admitting them to advanced work.

(6) Testing seniors and graduate students on their retention of the fundamentals of elementary physics and on their ability to apply these to problems some years after the completion of the elementary course.

(7) Rating applicants for departmental positions, such as laboratory assistant or examination reader.

(8) Estimating the relative abilities of different instructors. However, *this must be done with extreme caution*. The results of these programs indicate quite clearly that variability in scores due to differences in the abilities and backgrounds of the students is very much larger than variability in these scores due to differences in the teaching ability of instructors.

Comparability of results and forms.—The national men's percentiles for the 1939 tests and for the Revised Series tests[3] show achievement

[2] Am. J. Phys. (Am. Phys. T.) **2**, 129–148 (1934); **3**, 145–159 (1935); **4**, 153–166 (1936); **6**, 85–98 (1938).

[3] *Tables for the Interpretation of Scores on the Cooperative Physics Tests for College Students*, issued by the Cooperative

quite comparable to that of previous testing programs. The difficulties of the revised series forms have been equalized in advance of publication on the basis of data from earlier programs. Therefore the same norms apply to all four forms in this revised series.

Norms for women.—Beginning with the second annual program, separate abbreviated norms for women have been issued in accordance with the request of a large number of physics instructors. Generally speaking, women are less interested in scientific subjects than are men, and few women study physics with any professional objective. It is probably for these reasons that the scores of women students on the physics tests are not as high, on the average, as are men's, but tend with fair consistency to center at the 30th percentile of the distribution of scores for men. There is some evidence that the women physics students who take these tests are about equal to the men in their performance on the sound and modern physics tests, but their scores are inferior to the men's in mechanics, heat, light and electricity.

Norms for engineering students.—Students in engineering colleges and in schools and departments of engineering in universities should be somewhat superior to others in their work in physics, because their vocational choice indicates some scientific interest and background, and because they are studying the subject as a professional, rather than as a cultural, subject. In many colleges, in fact, a separate and more thorough course in elementary physics is offered to students in engineering courses than to students in other divisions or departments. Comparing the percentiles for engineering students on the 1938 tests and the revised series tests with the national men's percentiles on these tests, we see that the superiority of the engineering students is of the order of four or five percentile points.

Pre-study norms.—Separate pre-study norms were not provided with the original tests, as too few colleges returned pre-test results to warrant their compilation. Studies of the pre-test averages and of the average gains in achievement from pre-study tests to post-study tests indicated, however, that such norms would be valuable. By combining results for several years, it was possible to prepare adequate pre-study norms for the revised series tests.

Types of colleges.—Studies of achievement in the various types of colleges have been inconclusive. It would seem that the cases studied in the 1938 returns have not been numerous enough to prevent sampling errors, for the engineering colleges and their departments stand no higher in mean performance than the men's liberal arts and junior colleges. Only small numbers of cases were available from the teachers colleges and the agricultural and junior colleges. The median semester scores made by various groups of colleges do not differ noticeably from the national medians, except in the case of those reported by the teachers colleges, whose median semester scores were somewhat lower than the national norms in the first semester.

Variability in achievement.—The great variation in achievement among departments in different institutions, as well as among students in the same department, has been noteworthy. On the whole, the differences between departments are about half as great as the variation among all individual students. Variability among students in the same department, however, is about 80 percent as large as in the total group. The distribution of college averages on recent tests resembles that reported from the 1936–37 testing program.[4] On the first semester total, the averages of two colleges are below the 9th percentile of the distribution of scores obtained by all students taking the tests, and on the second semester total the average of one college is above the 91st percentile.

From such figures, considering also the great range of raw scores from the high percentiles to the low, two important problems relating to variability are evident. The first is concerned with the necessity for adjusting instruction to the large differences among students within the same institution. In large departments it is occasionally possible to schedule separate sections for the high-, average- and low-scoring students. Such sectioning, however, is only a partial solution to the problem, since wide indi-

Test Service with the cooperation of the committee on tests of the American Association of Physics Teachers.

[4] Am. J. Phys. (Am. Phys. T.) **6**, 85–98 (1938).

vidual differences occur even within these groups. Generally speaking, adaptation of instruction to individual differences is a problem that must be met by each instructor in each class, and an objective measure of the extent and nature of these differences is necessary for effective teaching.

The second problem concerns the variation in academic standards. Since college averages vary from below the 10th percentile to above the 90th, it is obvious that departments of physics in different institutions have entirely different purposes. One of the great values of objective examinations, provided as they are with accurate national norms, lies in their demonstration of the fact that academic standards of different institutions are not even approximately uniform. Elementary physics *should* be offered in different ways in different institutions if it is to have maximum value for all students. For example, the standards of a department of physics serving primarily the requirements of professional students in science and engineering probably should be quite different from those of a department serving mainly the cultural needs of students in general education. *The existence of comparable tests makes it less necessary than ever before to try to insist upon uniform academic standards* in all accredited colleges, since the transcript of grades is no longer the only available indicator of a student's accomplishment in physics.

Class in college.—One or two studies have dealt with the scores of students in the various college classes. The differences found were all small, and the only tendency was that the scores of sophomores exceed slightly the scores of those students in either lower or higher classes. Since the elementary course in college physics is usually a sophomore subject, this may merely indicate the probability of a slight positive relationship between ability in this subject and the tendency to take it at the usual time.

Conditioning Factors

A number of different factors in the backgrounds of the students have been shown to be significantly related to success in college physics, while others, seemingly equally important, have exhibited, upon investigation, no such relationships.

Gains in achievement and the effects of high school physics.—The reports of all the annual programs emphasize the measurement of growth in knowledge of physics through the use of both pre- and post-study tests. They also indicate that the presence or absence of high school physics in the student's background is the factor having the highest relationship to achievement in college physics. The reports have shown large and consistent differences in favor of the students who have had courses in physics in high school; and these differences are *at least as large on the post-study tests* as on the pre-study tests. If the principal influence of high school physics is assumed to be its direct contribution to the student's background, we should not expect this to be the case. Rather, the median scores of the two groups after studying a unit would probably tend to be much more similar than the median scores on the pre-tests. It is quite possible that high school physics gives the student a general background of considerable magnitude, but that this background must be amplified by further instruction in order that the student may possess the most effective working knowledge of the subject matter of physics at the difficulty level of these tests. On the other hand, it is likely that students who are interested in physics and who have superior ability along this line elect it in high school more often than those who do not possess such ability.

The studies also bring out the fact, previously noted, that many students, even in the groups which have not studied physics in high school, stand above the national post-study averages at the time they take the pre-test. Many others remain below the pre-test averages even after college instruction in the best departments. These facts emphasize again the importance of adapting instruction to the abilities and background of the individual student.

Time requirements.—The effect of different time requirements on achievement in college physics has been the subject of various studies. However, these studies have been inconclusive. On the basis of the 1939 returns, distributions of semester total scores were made for students in courses requiring 2, 3 or 4 hours of lecture and recitation per week. These data indicated no definite superiority for any group studied. A

similar study based on hours of laboratory work required per week was also undertaken. In this instance, the results showed a slight tendency toward achievement of higher scores on the tests by students in courses requiring larger numbers of laboratory hours.

A further study according to hours of credit has been made of the averages on the various tests for students in liberal arts colleges participating in the 1938 program. Here again there is no clear tendency for students in the longer courses to surpass those in the shorter ones. All in all, considering the studies made in previous years, as well as those made in 1938 and 1939, it has not been demonstrated that a long course produces any more actual achievement, of the types measured by these tests, than does a short course. If an instructor covers the units in *any* effective manner, the primary determining factors in the scores of the students would seem to be their own abilities, interests, backgrounds and efforts rather than the number or type of class periods devoted to the course.

Professional goal.—On a number of the test forms, spaces have been included in which the students were requested to state their professional goals. Studies of responses indicate that, in four different years, the levels of achievement in physics attained by groups expressing the intention of entering engineering, teaching, or medicine have tended to remain the same, although achievement on separate parts of the physics tests has fluctuated from year to year. The other groups studied, including students of business, agriculture, law and architecture, present relatively fewer cases, and the fluctuations are very marked from year to year. On the basis of data available, there seems to be a tendency toward the improvement of the achievement of students of agriculture, especially in the second semester. The achievements of law students shift about, as should be expected from the smallness of the number of tests taken by them.

Engineering, teaching and medical aspirants generally rank in achievement in the order named in all divisions of the tests except modern physics. In modern physics, students intending to teach retained the lead until 1937–38, when the agricultural and medical groups secured it. As pointed out before, one consistent finding from this and previous similar studies is that those individuals who intend to become engineers are superior to others with respect to scores on these tests. The difference between the percentiles for students in engineering colleges and departments and the general norms for men is not, however, as large as one would expect on the basis of these results. It would appear that the specific intention to become an engineer is a more definite indicator of superiority in physics than is enrolment in an engineering college or department.

Prerequisites.—Investigations concerning the effect of prerequisites on physics test scores seem to indicate that their importance is slight. An examination of the medians and distributions of semester total scores on the physics tests yields evidence of no real differences between scores of students in classes *requiring* high school physics and those *not requiring* this subject, or between scores of students in classes *requiring various amounts* of mathematics as a prerequisite. Students who have been required to have one, two or three years of mathematics appear to do little, if any, better than those in classes not having any definite mathematics prerequisites. Even in the case of high school physics, the students in courses requiring this subject as a prerequisite do not seem to be noticeably superior to those in other courses. This may possibly be due to the fact that high school physics is a prerequisite in only a small number of institutions, whereas a considerable majority of all students in college physics classes actually have had courses in this subject in high school. It would seem that prerequisites, as usually defined in terms of courses taken, are of little value, since there are such wide differences in standards and courses of study in various school systems. If students were required to exhibit competence on objective tests in all of the prerequisite fields, it seems highly probable that the differences in favor of prerequisites would be much larger than they appear at present.

Data from Questionary

Physics instructors who have used the tests, and particularly those who have used them for some years, are in a position to provide practical evaluations of both the tests and the programs

TABLE I. *Physics inquiry blank.*

1. What has been the general attitude of your students toward this testing program?*
   a. Exceedingly favorable. [5]
   b. Definitely favorable. [58]
   c. Mildly favorable. [47]
   d. Indifferent. [9]
   e. Noticeably antagonistic. [18]

2. What is your own attitude, as a teacher, toward these tests?*
   a. Exceedingly favorable. [26]
   b. Definitely favorable. [83]
   c. Mildly favorable. [17]
   d. Indifferent. [1]
   e. Noticeably antagonistic. [1]

3. How have these tests influenced your classroom teaching?**
   a. Called attention to large individual differences, and consequent need for adapting instruction definitely for students having different abilities, backgrounds and needs. [62]
   b. Indicated specific areas of strength and weakness in backgrounds of students, leading to changes in emphasis on the several major units. [43]
   c. Pointed out differences in effectiveness of various teaching methods tried out, leading to the improvement of instruction. [40]
   d. Tended to clarify instructor's ideas about objectives of the course, and to cause him to place greater emphasis on essential points. [45]
   e. Tended to cause instructor to emphasize points tested, at the expense of other equally important points. [10]
   f. Other influences. ....
   g. ....
   h. ....
   i. No appreciable influence. [11]

4. Are your post-study dates announced to students in advance?*
   a. Yes [118]
   b. No [12]

5. What part do the Cooperative Physics Tests (post-study) play in determining the final grades of your students?*
   a. Used as sole basis. [0]
   b. Given regular weight of percentage indicated (31%). [100]
   c. Used only to corroborate other scores. [22]
   d. Not used at all in determining grades. [3]

6. What other uses have been made of the post-study test results?**
   a. Providing information used in advising students to continue or drop further work in physics. [51]
   b. Prerequisite for entrance into intermediate courses in physics. [5]
   c. Sectioning students in intermediate courses. [1]
   d. Selection of departmental assistants. [16]
   e. As a test on general physics for students beginning graduate work. [17]
   f. Affording individual students a concrete method of evaluating their status and progress. [79]
   g. Other uses. [4] ....
   h. ....
   i. ....

7. How often do you give written tests or quizzes? (Estimate if necessary; do not count mid-term and final examination.)
   a. Daily. [4]
   b. Weekly. [30]
   c. Bi-weekly. [30]
   d. Monthly. [41]
   e. Two or three times a semester. [24]
   f. Never give written tests or quizzes other than mid-term and final examinations. [5]

8. What procedures do you use for motivating your students to do their best?**
   a. Report grades regularly. [64]
   b. Report Cooperative Test grades (post-study) regularly. [58]
   c. Report pre-test and post-study test results, and emphasize improvement. [20]
   d. Regular oral quizzes and recitations. [74]
   e. Regular written problems. [91]
   f. Regular written reading reports. [7]
   g. Regular written reports of laboratory work. [107]
   h. Individual counseling at start of course. [29]
   i. Individual counseling at mid-term. [55]
   j. Individual counseling at end of first term. [34]
   k. Other procedures: *weekly quiz* [3]
   l. *Frequent individual counseling* [14]
   m. ....

9. What test scores do you usually report to your students?**
   a. Pre-test raw scores. [24]
   b. Pre-test national percentile ranks. [18]
   c. Post-study test raw scores. [85]
   d. Post-study national percentile ranks. [78]
   e. No scores reported to students. [8]

10. What uses have been made of the pre-test results?**
   a. Sectioning students into appropriate classes. [2]
   b. Counseling students on the advisability of taking or continuing the course. [12]
   c. Indicating to the instructor the general status of the class at the start of each major unit of work. [35]
   d. Evaluating the effects of previous instruction in high school. [21]
   e. Selecting able students for special work. [15]
   f. Selecting deficient students for special attention. [26]
   g. Other uses: *review purposes* [1]
   h. *Indicate student's own progress and need of systematic study* [1]
   i. ....
   j. Pre-tests not used. [65]

11. How much more valuable do you find the national percentile norms sheet when this is issued in advance with the test shipment?*
   a. Very much more valuable. [54]
   b. Appreciably more valuable than when norms were issued the following summer. [42]
   c. About the same value as when norms were issued the following summer. [13]
   d. Very little value in any case. [7]

TABLE I. (*Continued.*)

12. How is your class time divided, approximately? (List percentages)
    a. Lectures by the instructor. [21]
    b. General class discussions. [13]
    c. Recitations and oral quizzes. [12]
    d. Special reports by students. [3]
    e. Lecture-demonstrations. [13]
    f. Moving pictures; slides. [3]
    g. Laboratory work by students (clock hr). [29]
    h. Examinations, tests and written quizzes. [69]

13. What types of test items do you feel to be most valuable? (Write in any additional types that you feel to be important, and then place a *1* before the most valuable type, a *2* before the next most valuable, etc. *Mark every one.*) Items
    a. Calling for definite knowledge of important facts. [4.3]
    b. Calling for definite knowledge of important principles and laws. [2.4]
    c. Involving numerical problems. [4.3]
    d. Calling for the use of mathematical concepts of types higher than those of first-year high school algebra and geometry. [6.1]
    e. Calling for the *interpretation* of facts or principles of physics, so that a student who *knows* the fact or principle may still miss the question if he cannot *use* it in a situation involving physical reasoning. [1.9]
    f. Calling for the application of facts and principles to practical situations met with in everyday life. [3.3]
    g. Calling for comprehension of the meanings of important technical terms. [4.9]
    h. ....
    i. ....
    j. ....

14. Data are desired which may help in answering the question of whether or not outstanding ability in the graduate school can be predicted on the basis of test results obtained during an elementary course in physics. If you have a graduate department in your institution, list here the names of the five outstanding graduate students, together with the college at which each of these students took his first course in college physics, and the year. Let the student *estimate* this last if he does not remember accurately.

* Check one.
** Check one or more.

on the basis of their experience. In order to take advantage of this valuable source of information and pooled judgment, the questionary shown in Table I was sent to at least one instructor in every institution that has used the tests recently. One hundred and thirty of these questionaries were returned in the relatively short time necessary for inclusion in this report. Summaries of the results appear in square brackets in Table I. The following numbered comments apply to the corresponding items on the blank.

1. Most of the students were thought *by their instructors* to have reacted favorably to the tests. This may or may not be an accurate gauge of actual student opinion.

2. The nature of the sampling would almost guarantee a group of instructors favorable—or at least not unfavorable—to the tests. The instructors appear definitely more favorable than they estimate their students to be.

3. By and large, the tests are affecting instructional methods favorably (Answers *a*, *b*, *c*). There might be arguments both *pro* and *con* on the merits of emphasizing the essentials as against covering a wider range of topics and problems (Answer *d*). Only 10 instructors report undesirable changes in emphasis (Answer *e*), often cited as a danger in theoretical discussions of comparable objective tests; and only 7 report that the tests have not influenced their teaching to any appreciable extent.

4. It is becoming the general practice, apparently, to announce the dates of important tests in advance.

5. Most instructors are using the results of these tests to improve the accuracy of their grades, giving them a reasonable weight in conjunction with other factors in arriving at final evaluations of the student's achievement. It is noteworthy that none of these 130 physics instructors have overestimated the value of the tests to the extent of using them as a sole basis for assigning grades. On the other hand, the 25 instructors who do not use the test results directly for this purpose are undoubtedly neglecting a valuable source of information. The average weight of 31 percent assigned to these tests in the evaluation of achievement seems a little conservative.

6. The major uses of the test results, other than in the assignment of grades, seem to be for purposes of student guidance (Answers *a* and *f*) rather than for administrative purposes (Answers *b*, *c*, *d*, *e*, *g*). These individual guidance uses are undoubtedly the most important ones.

7. While no control data are available, it seems fair to hazard a guess that the use of these tests has made instructors "quiz-conscious." Sixty-four report that they give tests bi-weekly or oftener. This represents measurement at shorter intervals than at the conclusion of each major unit of work.

8. The use of the tests for purposes of motivation is not as widespread as might have been supposed. The 20 instructors using the pre-study tests as well as the post-study tests for this purpose (Answer *c*) represent only one-fourth of those having both sets available, and only 58 of the 130 are using the post-study tests as instruments of motivation.

9. The great majority of instructors report scores to their students in one form or another. There still seems to be some preference for reporting raw scores rather than national percentiles, however. This may possibly be due to the fact that, until 1939, only the raw scores could be reported immediately, and many instructors undoubtedly felt that it was not worth the trouble to report national percentiles to the students some months after they had taken the tests.

10. The most important fact here is that half of the 130 instructors are using the pre-tests for some valuable educational purpose.

11. Most instructors appear to find the tests appreciably more valuable when supplied with adequate norms in advance of the program.

12. The instructors who use these tests estimate that they are devoting only 6 percent of their total class time to examinations, tests and written quizzes of all sorts, as compared to 12 percent devoted to recitations and oral quizzes, and 82 percent to strictly instructional activities. Most of them probably would agree that their recitations and oral quizzes possess both instructional and measurement values. In any event there is certainly no evidence that the instructors who use the tests are devoting any disproportionate amount of class time to measurement activities.

13. This ranking agrees quite well with the ranks that test experts would assign, except in the case of Answers *a* and g. Items measuring knowledge of facts and of technical terms have been found in practice to be among the very best of all types, provided, of course, that the facts and technical terms are really important. The low rankings given to these types of items by the physics teachers may possibly be due more to the influence of certain educational philosophers than to their own classroom experience.

14. Although the information supplied with regard to this item was too limited for statistical study or extensive statement of conclusions, those replies which were available and capable of being checked indicated the pronounced superiority on the Cooperative College Physics Tests, taken during the elementary physics course, of those individuals who were later designated as "outstanding students" in graduate physics study.

Appraisal of the Program

As the national college physics testing program enters its seventh year, it seems fair to say that it has not only justified its existence but that it has also made significant contributions to the development of physics teaching as an art and as a science. There is still much to be done in the way of improving the tests, and more in the way of improving their use. The credit side of the ledger, however, considerably outweighs the debit side. Among the values of the program, the following may be noted particularly:

1. It calls forcibly to the attention of physics teachers the large differences in ability and background among their students, and the consequent need for adapting instruction within departments, and even within classes, to these differences.

2. It points out the fact that different institutions have, and should have, different academic standards; and provides, at the same time, through their objective scores and national norms, an independent device for the evaluation of student achievement, thus obviating the need for standardization of instructional programs and grading systems.

3. It provides a somewhat different type of measure of student achievement from those employed heretofore, which, in combination with other measures, leads to increased accuracy of student appraisal.

4. It encourages, through the provision of comparable pre-study and post-study tests for each of the major units, the concentration of the instructor's attention on the actual improvement of each student in knowledge and grasp of the materials of elementary physics.

5. It furnishes objective evidence of value in the solution of a number of problems concerning the factors that affect ability and progress in physics.

6. It emphasizes the importance of individual personnel and guidance work with students, and provides valuable information and a useful point of departure for such work.

Among the things that still need to be done before the program can achieve maximum effectiveness, the following might be mentioned:

1. Point out the values of the program to the instructors who are still not acquainted with them, who still remain skeptical of them, or who still fear certain dangers. It should be emphasized that the objective tests are designed to supplement other instruments and methods of evaluation, *not* to supplant them, and that one of the major purposes of the program is to *reduce* standardization among courses rather than to encourage it. This is accomplished through the provision of comparable instruments of measurement independent of grade transcripts, so that comparability among the latter becomes less necessary than before. The existence of pre-program norms for the revised series tests should be noted particularly.

2. Acquaint a much greater number of instructors with the values of the pre-tests, and the advantages to be derived from measuring growth in accomplishment as well as end-accomplishment in elementary college physics, since a considerable part of the value of the program is lost to instructors who do not understand and use the pre-tests.

3. Bring to the attention of instructors the fact that the individual guidance uses of these comparable objective tests are as important as their administrative uses, or even their use in improving the accuracy of grades.

4. Make studies in various colleges of the validities of the tests and test items, so that factual data will be available for the information both of those instructors who are unduly skeptical and of those who are unduly enthusiastic concerning the values of objective examinations. Particular attention should be paid to the actual efficacy, in discriminating between good students and poor students, of the various types of items. The accumulation of data of this type will provide the only sure basis for the improvement of future editions of the tests.

In conclusion, the committee on tests wishes to offer its thanks to the hundreds of physics instructors who have continuously participated in the programs. The success of this project, both in helping physics teachers to improve their appraisals of student achievement and in exhibiting to all college teachers the values inherent in the cooperative approach to measurement problems, is due primarily to their generous support and their intelligent criticism.

The committee also wishes to acknowledge the assistance rendered by the staff of the Cooperative Test Service, particularly by Dr. Edward E. Cureton, formerly Research Advisor, and Dr. Richard E. Watson, Science Editor, in the preparation of this report.

The Committee on Tests of the American Association of Physics Teachers.—H. W. Farwell, Columbia University, Harvey B. Lemon, University of Chicago, Frederic Palmer, Jr., Haverford College, John T. Tate, University of Minnesota, A. G. Worthing, University of Pittsburg, C. J. Lapp, State University of Iowa, *Chairman.*

Physics of Deep-Sea Diving

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THE model to be described, with comments on points it suggests, will illustrate the sort of thing the demonstration lecturer in general physics can do, rather frequently, to stimulate and conserve a more alert interest. Whenever a student begins to think about the physical principles that are found in many newspaper items and magazine articles, the subject of physics is undoubtedly taking hold of him. Such an attitude is not too hard for the lecturer to arouse and promote in the majority of students. The only conditions are that he himself be alert continually to such opportunities, and be willing to make whatever extra effort may be needed to present effectively to the class his own reactions.

The Model

A household platform scale, "postal" type, of 25 lb capacity is adapted to read *pressures* instead of *forces*, as it would do in ordinary use. A special dial, drawn with India ink and reduced photographically (Fig. 1), is *calibrated* in four different units—pounds per square inch, atmospheres, depth in fresh water and depth in the sea—and is attached to the face of the scale over the original dial (Fig. 2). It permits, for a given deflection, four simultaneous readings of the corresponding pressure.

To deal with pressure, we must connect the force causing the deflection of the pointer with some particular area over which the force acts.[1] Two short lengths of cylindrical rod (wood dowel) are placed vertically with ends opposed. The lower segment is fixed to the platform; the upper one communicates flexibly with a horizontal lever. The cross-sectional area of each of

[1] The original calibration in pounds force can be retained advantageously in the new dial (which was not done in the present model) to emphasize the character of the definition of pressure.

the opposed faces is 0.100 in.[2]. By placing an object between the two pincer-faces, it can be subjected to any pressure within the range 0 to 250 lb in.$^{-2}$, with corresponding water depths from 0 to about 600 ft, a range that includes amply the present record of 420 ft attained by "rubber-clad" divers.[2]

Applications in Physics

The dial illustrates visually the relation between the *atmosphere* and the *pound per square inch* as pressure units. Although it is easy for anyone to memorize the statement, "One atmosphere equals 14.7 pounds per square inch," a visual and, especially, a tactual demonstration of pressures is helpful. A teaching aim should be to bring these rapidly memorized relations down from the nebular realm of "might be" to that of conviction in the world of everyday physical experience.

The dial is really one kind of *graphical representation* of direct proportionalities, and thus suggests the topics of *constants of proportionality* and *conversion factors* ("C.F.'s").

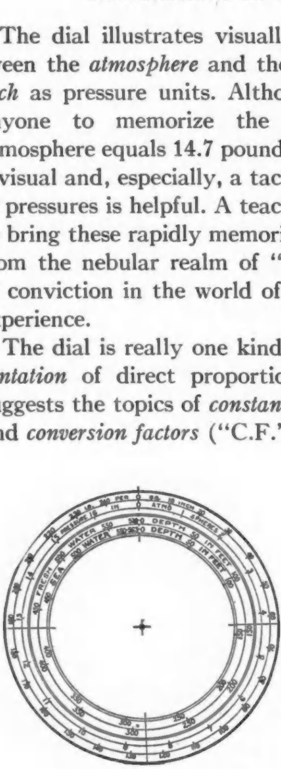

Fig. 1. Drawing for the special dial.

Between any two of the four calibration scales on the dial there is a constant of proportionality (C.F.) with a definite numerical value. Suppose we wish to convert a pressure of 200 lb in.$^{-2}$ to the equivalent depth of fresh water. Symbolically we may represent this process thus: lb in.$^{-2}\rightarrow h$ (ft, fresh water). The arrow represents proportionality, but also calls for the specific process of *conversion of units*. The C.F. here is 2.31. Thus 200 lb in.$^{-2}\times 2.31$ $=462$ ft, depth in fresh water. Let the student also verify the following values of C.F.: (1) lb in.$^{-2}\rightarrow$atmos, 0.0680 $=1/14.7$; (2) lb in.$^{-2}\rightarrow h$ (ft, sea water), 2.25; (3) atmos$\rightarrow h$ (ft, fresh water), 34.0; (4) atmos$\rightarrow h$ (ft, sea water), 33.2; (5) h (ft, fresh water)$\rightarrow h$ (ft, sea water), $0.975=1/1.026$. (Denominator means what?)

The dial, presented on a lantern slide, affords an excellent opportunity to discuss *pressure head* in terms of two different liquids, fresh water and sea water. Air pressures in pipe organs are usually given in terms of pressure head, with fresh water as the liquid and the inch as the unit. Barometer readings are in terms of, say, millimeters of mercury. The equation $p=\rho gh$ serves to show that we can use length to represent pressure, since it is proportional to the pressure. Applying the formula, we could easily add to the dial two additional scales giving pressures in cgs and gravitational units.

Some Practical Questions

We may ask and answer several practical questions:

(1) Given a certain depth in fresh water, find the depth in sea water for which the pressure is the same. The answer must depend on the two densities. Letting h_1, h_2 represent the respective depths, and ρ_1, ρ_2 the densities, we have $p=\rho_1 gh_1=\rho_2 gh_2$; an inverse proportionality exists between h and ρ, and $h_2=(\rho_1/\rho_2)h_1$. The ratio ρ_1/ρ_2 is the C.F. 0.975 in conversion (5) of the preceding section.

(2) For any given pressure, what is the *percentage difference* between the corresponding depths in fresh water and sea water? Choosing the depth h_1 in fresh water as 100 percent, we *define* the percentage difference as $[(h_2-h_1)/h_1]\times 100$. Note that the algebraic sign is just as important as the numerical value, for it indicates the direction in which this difference lies. When $(\rho_1/\rho_2)h_1$ is substituted for h_2, the expression for the percentage difference becomes $[(\rho_1-\rho_2)/\rho_2]\times 100$, a *constant* value independent of the depth. But this is the negative percentage difference of the two densities, since we are using fresh water as a basis of comparison. This conclusion may be extended generally: *where an inverse proportionality exists between two variables, x and y, and we are relating two cases, $x_1y_1=x_2y_2$, the percentage difference of the two x's is the negative percentage difference of the two corresponding y's.*

Since $\rho_1=1.000$, $\rho_2=1.026$ gm·cm^{-3}, the difference in the ρ's is $+2.6$ percent, while that in the h's is -2.6 percent, meaning that the density of sea water is 2.6 percent greater than that of fresh water, while for the same pressure the depth is 2.6 percent less. Thus, for every 100 ft depth in fresh water, the sea water depth for the same pressure is 2.6 ft less, or 97.4 ft. These figures emphasize the practical pressure-effect of this amount of density difference as a diver descends. The depth-difference for the same pressure in the two liquids is *directly* proportional

[2] Attained with a Craig-Nohl diving suit, December 1, 1937, in Lake Michigan. See Craig, *Danger is my business* (Simon and Schuster, 1938).

to the depth in either liquid. (Compare the two constants of proportionality.)

(3) At any assigned depth common to both liquids, what percentage difference exists between the two pressures? Here $h=h_1=h_2=p_1/\rho_1=p_2/\rho_2$, a *direct* proportionality between p and ρ. Then percentage pressure-difference is $[(p_2-p_1)/p_1]\times 100$ or $[(\rho_2-\rho_1)/\rho_1]\times 100$, so that at the common depth, whatever its value, the percentage pressure-difference is the same as the percentage density-difference, with no change in sign. (The conclusion can be extended at once to direct proportionalities generally.) Moreover, the pressure-difference is proportional to the depth (neglecting the very slight change in density of a liquid with pressure); let the student prove this, even though the fact may seem obvious, to offset the frequent tendency of the beginner to assume a direct proportionality in situations where it does not necessarily exist.

A Physiological Demonstration

By placing the finger in the pincers of the model (Fig. 2) and gradually pressing on the lever, one becomes aware in a very direct way of the magnitude of the pressure upon the entire surface of a diver's body when submerged to different depths, as read on the dial. Reference can be made also to other pressures, such as those in steam boilers and in automobile tires. One can easily endure as much as 250 lb in.$^{-2}$ so applied to the finger, although the smooth-faced pincers leave momentary marks on the skin. In underwater diving there is this difference, that the *external pressure* is applied over the entire surface of the diver's body.

One may ask, by what percentage is the pressure on top of a diver's helmet greater than that at his feet, assuming a vertical distance of 7 ft between these two points? Notice that the answer depends on the depth; also, one may refer to the problem in hydrostatics of the torque effective on a water gate hinged at the top, the hinge being below the water level on the upstream face of the barrier.

The finger in the pincers is quite similar to a liquid inside a closed sac that is being pressed upon locally from opposite sides (Pascal's principle), although the bone must play some part as an *elastic solid*. The pain felt may be due largely to the sharp curvature where the slightly beveled edges of the pincer-faces contact the skin. Doubtless the diver does not feel such localized pain. Nevertheless, the pressure upon him at a given depth is that indicated by the dial of the model.

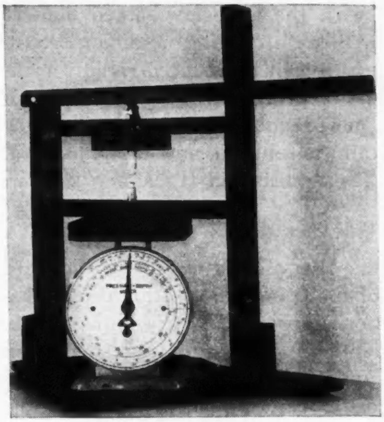

Fig. 2. The demonstration model.

Deep-Water Diving

"Rubber-clad" is a term descriptive of a diver breathing air at the same pressure as that of the water surrounding his rubber suit. Because of the increasing air pressure as he descends, his blood and watery tissues dissolve more and more nitrogen; the oxygen in the air he breathes is used up in oxidation processes in the blood. If the diver remains at a given depth long enough, his system will become saturated with nitrogen in solution. This saturation is due to the pressure, regardless of whether it is applied by surrounding water or by the air in a compression chamber where he might be placed. The diver's suit *is* a compression chamber, since it contains air at a pressure approximately equaling that of the surrounding water.

At atmospheric pressure, a man's body holds about 1 l of nitrogen in solution. This amount saturates his system at that pressure, as far as this gas is concerned; no more will be dissolved unless the pressure is raised. The effect of increasing the pressure of the air in which the diver is immersed (inside his rubber suit) is entirely similar to charging water with carbon dioxide in making "carbonated water." We know that when the pressure on a tank of carbonated water is released, bubbles of carbon dioxide form quickly. A diver who has been for some time at a considerable depth is, in effect, "a charged tank of nitrogen water."

But with the nitrogen-charged man there is this further interesting and important fact: the pressure can be reduced rapidly to as little as one-half its value without the bubbles forming. At the new, reduced pressure the diver's system still retains in solution, now *supersaturated*, 1 l of excess gas as measured at the new pressure.

Rates of Ascent in Diving

Assume that a diver works in fresh water at a depth of 100 ft long enough to attain a degree of saturation corresponding to complete saturation at a depth of 54 ft. To what depth may he ascend quickly without danger of "compressed-air illness"? The problem is solved by dividing the saturation pressure by 2 and finding the corresponding new depth. Thus, including, of course, the atmospheric pressure on the water surface, and expressing pressure in feet of fresh water, we have $\frac{1}{2}(54+34)$ or 44 ft. Subtracting atmospheric pressure, we have 10 ft, the depth to which he may safely ascend at one time. One might think that, being only 10 ft from the surface, the diver could with little risk be brought all at once to the top. But if that should be attempted, there is danger of reaching the "breaking-point," which is the rather critical pressure at which bubbles will form, with bad results to the diver.

In the old days of diving these facts were unknown. Methods of gradual ascent were therefore quite arbitrary. For example, the old-time diver starting to ascend from the 100-ft depth, under the conditions of the problem just stated, might pause for a time at 70 ft. But instead of his system giving off nitrogen from solution, it would continue to dissolve the gas, because the diver would be saturated for the lesser depth of 54 ft only; he might with safety have come directly from the 100-ft level to within 10 ft of the surface. Thus these earlier methods of gradual ascent were unscientific and not a sure preventative of the "bends," a disease enveloped in uncertainty and mystery. Progress in learning the true character of the "bends" appears to have been made since 1900, although Paul Bert had concluded about 1880 that slow ascent was necessary to avoid the disease.[3]

[3] See J. S. Haldane, "Hygiene of work in compressed air," Eng. Rec. **58**, 469 (1908).

Compressed-Air Illness

Compressed-air illness ("bends," or "caisson disease") is prevented by a sufficiently slow decompression to permit the necessary amount of excess gas in solution to pass off physiologically by way of the lungs. When a person who has been under relatively high pressure for some time is too rapidly decompressed, the formation of bubbles in his blood and tissues may be fatal and in any event calls for emergency treatment. The basic treatment is recompression of the patient as soon as possible. This means putting him under enough pressure to redissolve in his blood and tissues the gas in the bubbles. Recompression has two beneficial effects: (1) the immediate reduction in volume of the bubbles, probably very closely in accordance with Boyle's law; and (2) the redissolving of the gas, causing the bubbles to diminish still further in volume and to disappear.

F. Meier,[4] an experienced diver, mentions considerable variation among divers as to susceptibility to the "bends." However, he gives in illustration an incident that occurred in 1910, apparently with no measurements in the scientific sense, correlating depth and time; hence, enough actual difference in conditions may have been present to explain the differing effects observed in the individuals. Another suggestion is that individuals may differ appreciably in their retentivity of the supersaturated nitrogen.

Meier mentions the beneficial effect of exercise soon after a too rapid ascent. This is understandable; the exercise will accelerate both circulation and breathing, thus promoting the throwing off of the excess nitrogen by way of the lungs. It would seem that the amount of water drunk under such circumstances and the elimination of moisture from the system should be factors of some importance.

Decompression Tables

The public hears of deep-sea diving as a daily event, but usually draws no very clear distinction between work at 200 ft and at 100 ft. Actually the difference is large. For example, if

[4] Meier, Saturday Evening Post (1939): June 10, p. 5; June 17, p. 10; June 24, p. 23. The last article contains the most data of interest here.

the diver works at 50 ft for 40 min, the total time to bring him to the surface is less than 10 min. But if he works for the same length of time at 100 ft, the ascent takes more than 30 min, and at 200 ft, over 2 hr. The time of descent is relatively short.

Decompression tables afford data that permit as rapid an ascent of the diver as practicable without danger of compressed-air illness. In addition to preventing bubbles and saving the diver's valuable time, especially at greater depths, the tables keep at a minimum his exhaustion, which is increased during the ascent, due to cold, the handicap of heavy equipment and mechanical battling with the water currents, which when present can be very powerful. Although decompression tables are somewhat complicated, the main principles and facts governing them are quite concisely stated:

(1) The adult human body under the usual pressure of 1 atmos holds in saturated solution in the blood stream and watery tissues about 1 l of inert gas, practically all of it nitrogen.

(2) At constant temperature, the solubility of a gas in a liquid which does not act chemically upon it is proportional to the pressure (Henry's law).

(3) The time-rate of dissolving of a gas in the body under increased pressure is a known logarithmic curve, several hours being needed for saturation to be reached. The degree of saturation for a given immersion time is therefore known, and hence also the pressure and depth at which the solution of gas in the body at the time would be saturated.

(4) Compressed-air workers can remain for any length of time at a pressure of as much as 2 atmos and come at once to the surface without contracting the "bends." An abrupt drop in pressure greater than this can cause the disease.

(5) The human body exposed for some time to any pressure greater than 2 atmos can tolerate a rapid drop in the pressure to as little as half that at which gas saturation exists.

J. S. Haldane[3] argued from statement (4) that, if at 1 atmos, or just past the *saturation pressure*, it is possible for the body to hold 1 l of gas as a maximum in *supersaturated solution* without bubbles forming, then the same probably would be true as a general principle, whatever the saturation pressure, and that pressure, whatever its value, could be halved rapidly with safety. This would mean that a diver saturated at 4 atmos (about 100 ft) could come directly to a pressure of 2 atmos (about 33 ft); or for saturation at 10 atmos (about 300 ft, rapid ascent could be made to 5 atmos (130 ft). Tests upon animals have proved the correctness of this principle; namely, (5), which is now basic structure in all diving tables.

With maximum supersaturation of the body the situation is as though 2 l of gas at the saturation pressure had been crowded into a 1-l container provided with a valve arranged to prevent the excess liter from escaping, except with relative slowness, unless the surrounding pressure is further reduced; then the valve will open wide, reducing rapidly the pressure inside the container, which corresponds to what happens when bubbles form in the diver's blood stream and tissues. The valve in this illustration represents whatever physico-chemical mechanism is responsible for the phenomenon of "supersaturation" of a liquid with a gas. However, nitrogen dissolved in the body hardly offers so simple a situation as that where the solvent is a homogeneous liquid.

Applying the principle of abrupt or "rapid" ascent from a depth where the diver is saturated to that depth where the total pressure upon him has been reduced to half its first value, we have $\frac{1}{2}p_2 = p_1$ or, in terms of pressure heads, $(h_2+B) = \frac{1}{2}(h_1+B)$, where B is atmospheric pressure; whence $h_2 = \frac{1}{2}(h_1 - B)$, the shallowest depth to which the diver may rise rapidly with safety. The last formula illustrates a linear relation that is not a direct proportionality. "Saturation depth" is not necessarily the actual working depth of the diver; it depends on both working depth and time spent there. Some corresponding values of h_1 and h_2 are seen to be: 500 ft, 233 ft; 100 ft, 33.4 ft; 90, 28.4; 53, 10.0; 33.2, 0, or water surface. As stated previously, nitrogen saturation of the diver at any depths less than about 33 ft in sea water will not prevent bringing him at once to the surface. The maximum safe percentage change (a decrease) in depth is

$$\Delta h = [(h_2 - h_1)/h_1] \times 100 = -50(B/h_1) - 50,$$

a linear relation between Δh and the reciprocal of h_1, the saturation depth.

To save time, one would want to bring the diver as near to the surface as possible at the first "step" in the ascent. It is not, however,

necessary for him to stay at this new depth until enough nitrogen is thrown off through his lungs, without bubble formation in his blood stream, to reduce his state of supersaturation to one of saturation for the new depth. But his depth change in any step in the ascent must not exceed that permitted by the breaking-point rule, which asserts that rapid pressure reduction must not be more than about one-half of the pressure for which he is at the time saturated, regardless of his actual depth. For example, a diver working in sea water at 53 ft, or any greater depth than this, until he is saturated for a depth of 53 ft, can be brought at one step to a depth of 10 ft. He would then need to stay at this new depth only long enough to attain saturation for a depth of 33 ft; then, without further delay, he can be brought directly to the surface.

In this manner a set of decompression tables may be calculated, if for different depths there are known (1) the rates of solution to the point of saturation, and (2) the maximum rates of pressure reduction without the formation of bubbles. From such a set of tables[5] we select the examples in Table I. Working for 2 hr at 40 ft in sea water, a diver may come directly to the surface in a single step; after 4 hr at that depth he stops at the 10-ft level for 5 min, then comes to the surface of the water. After 2 hr at 55 ft, he would come to the surface in 3 steps, stopping at 20 ft for 5 min, at 10 ft for 15 min. After 1 hr and 20 min at 250 ft, he would need to be brought to the surface in 11 steps, requiring $4\frac{3}{4}$ hr for completion of the ascent.

When a diver has rapidly ascended from deep water ("heavy water," in diving parlance), where he has been for some time, to the surface in a single step, he must immediately either be submerged again to a depth corresponding to half the pressure from which he came abruptly, or be put into a decompression chamber and subjected to that pressure and then gradual decompression according to the tables as they would be used in actual ascent.

"Helium Breathing"

For breathing purposes by compressed-air workers, the substitution of a helium-oxygen mixture for ordinary air is another important advance made in diving technic in recent years. The bad aftereffects of working in compressed air are due, as we have seen, to inert gas dissolved in the body under the extraordinary pressure. Eventually the question was raised whether some other inert gas might not be substituted for nitrogen in an oxygen mixture to reduce the threat of the "bends," to increase working depth and decrease time of ascent. Such

[5] *United States Navy decompression tables for divers.*

Table I. *Illustrative examples from decompression tables.*

Working depth (ft.)	Time under water meas'd from surf. (min.)	Step in ascent (No.)	Direct ascent to depth (ft.)	Stop (min.)	Total time of ascent (min.)	Total time ascending between stops (min.)	Av. speed of ascent (ft./min.)	
							Over-all	During actual ascent
40	120	1	surf.	—	1.5	—	26.6	26.6
	240	1	10	5				
		2	surf.	—	6	1	6.7	40.0
55	30	1	10	5				
		2	surf.	—	7	2	7.9	
	120	1	20	5				
		2	10	15				
		3	surf.	—	22	2	2.5	27.5
66	10	1	surf.	—	2	—	33.0	
	20	1	10	5				
		2	surf.	—	7	2	9.4	
	45	1	20	3				
		2	10	10				
		3	surf.	—	15	2	4.4	
	130	1	20	10				
		2	10	20				
		3	surf.	—	32	2	2.1	
	210	1	20	10				
		2	10	30				
		3	surf.	—	42	2	1.6	33.0
250	8	1	90	2				
		2	80	2				
		3	70	3				
		4	60	5				
		5	50	7				
		6	40	10				
		7	30	10				
		8	20	15				
		9	10	15				
		10	surf.	—	73	4	3.4	
	15	1	90	2				
		2	80	3				
		3	70	5				
		4	60	7				
		5	50	10				
		6	40	10				
		7	30	15				
		8	20	20				
		9	10	30				
		10	surf.		106	4	2.4	
	80	1	100	10				
		2	90	15				
		3	80	20				
		4	70	25				
		5	60	30				
		6	50	30				
		7	40	35				
		8	30	40				
		9	20	40				
		10	10	40				
		11	surf.	—	289	4	0.9	62.5

a gas is helium.[6] It is used to replace the nitrogen of the air in about the same volume proportion with the oxygen. In experimentation with animals, the results were favorable.

Helium and nitrogen may be compared numerically as to four different properties, all to the advantage of helium for the present purpose. Expressing each property comparatively in the form of a ratio, with the value for nitrogen in the denominator, we have: density, 1/7; solubility in water, 3/5; diffusion rate, 5/2; molecular weight, 1/7, facilitating escape through the lungs. One may also ask: what ratio of average velocities of molecules of the two gases at a given temperature is predicted by the principle of equipartition of energy?

A test in 1937 of helium breathing by men is related by Craig.[2] He and a companion acted as human guinea pigs under a pressure corresponding to 100 ft of water. The results were as expected, without bad aftereffects. The superiority of helium-oxygen "air" to ordinary air for diving and other compressed-air operations has been confirmed by such experiments.

Hardships Experienced by Divers

At the 1938 Annual Exhibition of the Physical Society of London, Captain G. C. C. Damant, R. N., lectured on the subject of diving.[7] He referred to Haldane's work,[4] the decompression tables based upon it, and how these tables have been extended for greater depths.

Damant related a peculiar diving accident, which illustrates vividly certain principles of physics. A sunken ship contained gold coins in the bottom of its hold. To retrieve them a hole was cut through the side of the ship near where the treasure lay. Attaching his line not far from this place, the diver found it necessary to crawl into the opening headfirst and remain with feet higher than head while he scooped the gold into a container. This caused air inside his suit to rise to his legs and feet, rendering them more and more buoyant, and making control of his movements increasingly more difficult. On one trip into the vessel he remained too long, and on emerging found the upward pull at his feet so strong that he could not right himself. He lost his hold and corklike was swept toward the surface. Long before he arrived there, however, his upward motion stopped when that part of the line between himself and its anchorage on the bottom became taut. He was thus tethered upside down and helpless. The telephone at the surface brought his faintly uttered but desperate cry for help, with the meager information that his suit was leaking. The diver immediately sent to his aid, knowing nothing of the true nature of the accident, followed the unfortunate diver's life line to the bottom but could see no one. Then discovering the part of the line leading up into the darkness above, he realized the situation. He had to act quickly. He cut the line from its anchorage. But his own line was now tangled with that of the man he was rescuing. Observers at the surface said later that both divers shot up out of the water like porpoises, so great was the buoyant effect of the expanded air inside the diving suits. (Compare the expansion of air bubbles as they rise in a jar of water.) The rapid drop of pressure to atmospheric would have been bad for both men, but this problem was met by putting the rescued diver into the decompression chamber while his rescuer immediately submerged again.

Damant's opinion was that underwater chambers of various designs permitting the diver to work for an indefinite length of time at greater depths, but at atmospheric pressure, and using contrivances such as mechanical arms and hands for doing the actual work, are not yet a complete success from the practical standpoint, even though their mechanical fingers may be able to "pick up a pin." A great advantage the rubber-clad diver has is that his arms and hands are free for the underwater work.

According to Damant, an unfailing characteristic of a certain deep-sea diver was that whenever he was brought to the surface and his helmet removed, he broke out with a steady barrage of talk, and the only way to stop it was to clamp on the helmet and send him down again. The American Beebe who had descended to depths even below 3000 ft in the bathysphere, a fortress he devised against the sea-water pressure, was the only deep-sea diver who, to Damant's knowledge, would ever admit there were any further ocean depths left to explore.

A diver's movements under water are slowed down and made more difficult due to the water resistance, particularly if currents exist, and to the weight and cumbrousness of his suit with its lead weights. Relatively intense cold may be an adverse factor. He may need to make great muscular effort while under constant nervous tension owing to unusual working conditions and the ever present danger of something going wrong. Even after stopping work at a given depth, he may yet have before him a protracted ascent with a strenuous battle against cold and currents bringing additional exhaustion. We must admit he has courage.

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[6] For a discussion of the origin of the suggestion to substitute helium for nitrogen, see Hildebrand, Science **65**, 324 (1927). See also *Reports of Investigations*, U. S. Dept. of the Interior, Bur. of Mines, Serial No. 2670, Feb., 1925, by Sayers, Yant and Hildebrand, for an account of the early developmental work and tests on animals.

[7] Damant, "Diving in deep water and shallow," J. Sci. Inst. **15**, 56–57 (1938).

Applied High Voltage Electrostatics

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The following experiment is devised to demonstrate basic electrostatic principles quantitatively with apparatus that finds practical application in modern electrostatic research. The experiment is given to physics majors in the junior physics laboratory at the Massachusetts Institute of Technology. After having previously studied the discussion, three students are able to take all the required data in a three-hour laboratory period. The curves presented are drawn from students' data.

1. Field Calculations

Many rough electrostatic field calculations involving high voltage terminals can be based on the approximation of a spherical terminal centrally located in a spherical shell. The field intensities E_1, E_2 and E at the two surfaces (Fig. 1) and in the intervening space, respectively, are related by the equations

$$Q = E_1R_1^2 = ER^2 = E_2R_2^2,$$

where Q is the electric charge stored on the terminal. The field intensity in the space is thereby related to that at the surface of the terminal by the equation $E = E_1(R_1/R)^2$. Integrating this expression from R_2 to R_1 gives the potential V of the terminal; that is, $V = E_1(R_1/R_2)(R_2 - R_1)$. To find the maximum potential which the terminal will support, replace E_1 in this expression by E_0, the dielectric strength of the medium surrounding the terminal.

In connection with voltage measurements, it is sometimes necessary to calculate the field intensity E_2 at the surface of the shell surrounding the terminal. The expression for it in terms of that at the surface of the terminal is $E_2 = E_1(R_1/R_2)^2$ or, in terms of the terminal potential, $E_2 = VR_1/R_2(R_2 - R_1)$.

These elementary equations are useful in connection with designing the external structure of electrostatic generators and calculating the necessary sensitivity of associated electrostatic voltmeters.

2. The Belt Electrostatic Generator

Any high voltage generator requires a terminal of large radius of curvature well insulated from ground. The essential difference between various types of generators lies in the method used in transferring charge to the terminal. A common form of belt electrostatic generator employs a spherical terminal mounted on an insulating tubular column. Charge is transferred to the terminal by means of a wide conveyor belt (or belts) operating inside the column in a region maintained at low humidity.

The insulating belt is charged at the grounded end by means of a row of points distributed across the face of the belt opposite the pulley or other grounded electrode, and maintained at a sufficient voltage to produce corona. Charge of one sign is driven by the electric field from the corona region to the surface of the belt and then carried by the belt to the terminal. Inside the terminal another row of points sprays sufficient charge of the opposite sign to discharge the incoming run of belt and to charge the outgoing run with opposite sign. Various arrangements for belt charging are discussed in references 1 (a), (b) and (c).

Static charge on a moving belt constitutes an electric current. The magnitude of the current depends upon the rate at which belt area enters the terminal and upon the surface density of charge on the belt. The maximum value of the surface density of charge is $\sigma_0 = E_0/4\pi$. Since the belt charge may be considered to be bound to one side of the belt at the pulley but distributed on both sides while the belts are free in the columns, the surface density of charge on *each surface of*

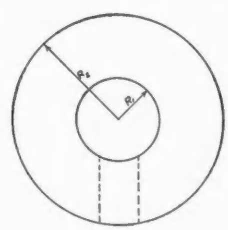

Fig. 1. Simplified geometry for high voltage terminal and housing.

each run of belt after leaving the pulley is

$$\sigma_0/2 = 1.32 \times 10^{-9} \text{ coul/cm}^2,$$

if E_0 is taken as 30,000 v/cm in the case of air under atmospheric pressure. Then the current delivered to the terminal will be

$$I = 2\sigma_0 WS = 5.30 \times 10^{-9}\ WS \text{ amp},$$

where W is the belt width (cm) and S is the belt speed (cm/sec). Currents realized in practice are only 60 to 70 percent of this calculated value, due to the fact that the realizable electric intensity is about two thirds of the ideal value. For example, a belt 1 ft wide and of speed 6000 ft/min delivers 300μa to the terminal. Since the belt tends to lose charge by corona in its passage through the column, the delivered current per belt may be increased by installing several belts with adjacent runs carrying opposite charge, by decreasing the separation between runs, or by closely shielding each run from its surroundings.

Steady voltages below the sparking value are maintained on the terminal by balancing the input current against the current drained from the terminal due to insulator leakage and corona. This can be accomplished either by varying the voltage applied to the spray points to control the charging current, or by mounting a set of corona points between the terminal and ground to control the current drain.

3. Voltage Measurement

The terminal voltage can be measured by the standard voltmeter method using a very high resistance and a microammeter, or by means of a spark gap, or by electrostatic methods which actually measure the electric intensity at some point in the space about the terminal. Using the electric intensity as a measure of terminal potential involves the acceptance of a well-established theorem: If in a *fixed* system of *conducting* bodies all potentials are increased by a given factor, then all electric intensities will be increased by the same factor. To obtain reliable results by this method it is necessary, therefore, to avoid the motion of conductors or the presence of dielectrics which might affect the electric intensity to be measured.

A. High resistance voltmeter

The simplest and most common arrangement for measuring moderate voltages is the high resistance-microammeter method. The only requirements are a high resistance of known value and a meter of known current sensitivity. However, there are certain difficulties with this method which become increasingly serious at higher voltages:

1. The composition resistor which must be used to obtain high resistance values is subject to uncertainties due to temperature, polarization, changes with time and a nonlinear current-voltage characteristic.
2. The collection of corona current along the resistor from the surrounding space may make the product IR an incorrect measure of terminal voltage.
3. Mechanical construction of the resistor unit becomes impracticable at very high voltages.
4. Sparks along the resistor are likely to damage it.

B. Spark gap voltmeter

The spark gap voltmeter has been used considerably in the past for the measurement of high voltage. This instrument depends on the fact that the voltage necessary to break down a given air gap between clean regular surfaces is very reproducible, provided that the air is dust-free and that its density is taken into consideration. A very complete discussion of spark gap measurements is given in reference 2(a).

The sphere gap voltmeter consists of a pair of spheres well removed from surrounding objects and so mounted that their separation may be smoothly controlled and accurately measured. The voltage at which sparking will occur depends on the gap distance, the sphere radius and the air density. The influence of these factors is indicated in the following equations.

The sparking voltage between spheres at standard air density is

$$V = eX/f(X/R) \text{ kv},$$

where $e = 27.2[1 + (0.54/\sqrt{R})]$ is the apparent sparking value of the electric intensity at the surface of the spheres, R is the radius of the spheres (cm), X is the gap distance (cm) and $f(X/R)$ is a complicated function of X/R (tabulated in Table I for the case of one sphere grounded).

Standard density ρ_0 is that density of air corresponding to 25°C and 760 mm of mercury.

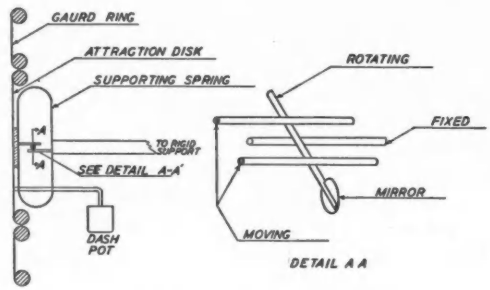

FIG. 2. Attracted disk voltmeter.

The sparking voltage at other densities ρ is quite accurately proportional to the relative density ρ/ρ_0. The complete expression for the sparking voltage at any density may then be written as

$$V=27.2\left(1+\frac{0.54}{\sqrt{R}}\right)\frac{\rho}{\rho_0}\frac{X}{f(X/R)}\text{ kv.}$$

The spark gap voltmeter has several disadvantages:

1. The voltage is destroyed at the instant that it is measured.
2. The voltage must be maintained steady and the spark gap decreased very gradually to the sparking value, to obtain an accurate voltage measurement.
3. Temperature and atmospheric pressure must be taken into consideration.
4. Smooth spheres and dust-free air are essential.
5. The spheres become large and unwieldly for very high voltages.

C. Attracted disk voltmeter

The attracted disk voltmeter (Fig. 2) is an electrostatic instrument which was developed very early for the absolute measurement of voltage. It depends upon the electrostatic attraction between parallel charged plates. The principle can be applied to determine the potential of an isolated terminal[1d] by mounting a grounded disk in the surface of the outer shell housing the terminal. If the disk is mounted in a friction-free aligning support and restrained by a weak spring, its displacement is a measure of the electric intensity at the surface of the shell.

The force acting on a grounded conducting disk due to the electric intensity at its surface can be calculated in the following way. The field induces a surface density of charge $\sigma=E/4\pi$. The force acting on an element of surface is given by $dF=\sigma E'dA$, where σdA is the charge on the element and E' is the contribution to the electric intensity at the element from charge in the surface outside the element. It can be shown[3] that $E'=\frac{1}{2}E$, so that

$$F=\tfrac{1}{2}\sigma AE=AE^2/8\pi.$$

From this it appears that the attractive force on the disk is proportional to the square of the electric intensity at its surface.

If the constant f of the restraining spring were known, the displacement of the disk could be used to calculate the electric intensity at its surface, since

$$E=(8\pi F/A)^{\frac{1}{2}}=(8\pi fx/A)^{\frac{1}{2}}.$$

Then, if the geometry were very simple, this electric intensity could be used to calculate the potential of the terminal. However, the general practice is to determine the calibration constant K in the attracted disk voltmeter equation,

$$V=K\sqrt{x},$$

by applying a known voltage to the terminal.

The disadvantages of the attracted disk voltmeter are:

1. The squared scale for the displacement as a function of voltage. This makes it necessary to use calibrating voltages of the same order of magnitude as the operating voltages, and limits the useful voltage range of the instrument.
2. The lack of sensitivity. In order to obtain a readable deflection on the scale for electric intensities ordinarily encountered, it is necessary to use a weak restraint on the disk and a large magnification in the optical system. The resulting vibrations and unstable zero seriously limit the accuracy of reading.

D. Generating voltmeter

The generating voltmeter is a more recent type of electrostatic voltmeter which avoids the

TABLE I. *Values of $f(X/R)$ for one sphere grounded.*

X/R	$f(X/R)$	X/R	$f(X/R)$
0.1	1.03	1.2	1.51
0.2	1.06	1.4	1.62
0.4	1.14	1.6	1.73
0.6	1.22	1.8	1.85
0.8	1.31	2.0	1.97
1.0	1.41		

squared scale. This instrument consists essentially of an electrostatic alternator and a device for measuring the generated current. In the alternator (Fig. 3) a grounded rotor plate is mounted in front of a grounded stator plate on a shaft extending through a small hole in the stator plate to a synchronous motor. Alternate sectors of the stator plate are insulated from the remainder of the plate and are connected to ground through a high resistance to provide an input voltage for the amplifier. The rotor alternately exposes these sectors to the high potential

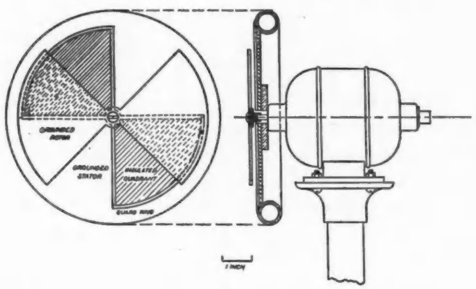

FIG. 3. Electrostatic alternator for generating voltmeter.

terminal and then covers them, thereby alternately causing charge to be induced on their surface and then permitting it to return to ground.

The alternator thus generates an alternating current having a square-topped wave form and a value (either peak or root-mean-square) given by

$$I = Q/t = \frac{(EA/4\pi)}{2f} = EAf/2\pi \text{ stat-amp},$$

where E is the electric intensity (stat-volt/cm), A is the sector area (cm^2), $f[=ns]$ is the frequency of generated current (cycle/sec), n is the number of sectors and s is the rotor speed (rev/sec). For convenient use the expression for the current may be written as

$$I = 1.77 \times 10^{-13}\, EAns \text{ amp},$$

where E is expressed in volts per centimeter.

If the alternator is located in a strong field, the output current can be rectified, either by a commutator mounted on the rotor shaft or by vacuum tubes, and applied to a galvanometer. A more flexible and sensitive instrument is obtained by the arrangement shown in Fig. 4, which includes a range selector, amplifier, rectifier and portable d.c. output meter.

The voltmeter equation in this case is $V = KI$, where V is the terminal potential and I is the direct current output. The calibration constant is again determined by applying a known voltage to the terminal. In this case, however, the linear scale and the wide range introduced by the range selector make it practicable to obtain an accurate calibration through the use of a relatively small calibrating voltage. Any question concerning the linearity of the instrument can be answered by mounting an insulated plate in front of the alternator with a spacing of about 1 cm and applying a wide range of known electric intensities by the use of relatively small voltages. A generating voltmeter installation is carefully analyzed in reference 4(c).

The generating voltmeter is subject to those limitations mentioned in the first paragraph of Section 3 as affecting any device that measures electric intensity. In particular, this voltmeter is sensitive to the presence of charged or polarized insulating surfaces in its field of view. The effect is serious only at very low humidities, and is minimized by mounting the alternator near the terminal and by keeping insulating surfaces at a distance.

4. VOLTAGE BREAKDOWN IN THE INSULATING MEDIUM

The limitation in the voltage which can be applied to a terminal is determined by the geometrical arrangement of the terminal and

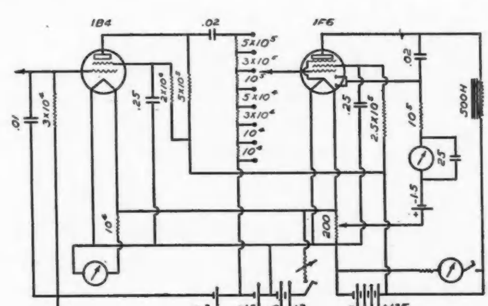

FIG. 4. Generating voltmeter amplifier with d.c. linear output circuit.

housing, and by the breakdown strength of the insulating medium. The proper consideration of these factors is part of the problem of high voltage generator design.

An examination of the expression for V in Section 1 indicates the influence of geometry in the case of concentric spheres. It is apparent, for example, that the terminal should be large and smooth to make R_1/R_2 large, and that the spark gap R_2-R_1 (Fig. 1) should be large. To determine the proper apportioning of these factors in a housing of fixed size, set $dV/dR_1=0$ and solve for R_1. This yields the optimum ratio, $R_1/R_2=\frac{1}{2}$, for concentric spheres. Similar calculations for concentric cylinders give for the optimum ratio, $R_1/R_2=1/2.718$.

If the insulating medium surrounding the terminal is air under atmospheric pressure, the breakdown strength is approximately 30,000 v/cm under ideal conditions, and more nearly 20,000 v/cm under practical conditions with large apparatus. For pressures other than atmospheric, the breakdown strength is quite accurately proportional to the pressure in the approximate range 0.001 to 10 atmos. If other gases, such as nitrogen and carbon dioxide, are used instead of air, little variation in breakdown strength is found. However, in the case of certain vapors, notably carbon tetrachloride and Freon, there is a considerable increase in breakdown strength, Freon having three times the dielectric strength of air. The insulation strength of gases and vapors is treated in reference 5(b), (c) and (d). The compactness attainable by generators employing compressed gas for insulation is indicated in reference 1(c).

Transformer oil, in spite of its high dielectric strength, is not satisfactory for the insulation of d.c. voltages in large apparatus. Migrating charges set up currents in the oil, which produce severe churning and constitute a considerable current drain on the terminal.

Voltage breakdown in gases may occur either as sparking or as corona. In the case of a spark the terminal is virtually grounded and drained of its current in a period of the order of a few microseconds. The spark then disappears, permits the terminal voltage to build up to sparking value and again shorts the terminal. In the case of corona limitation, the breakdown current increases smoothly with voltage (usually as the square of the voltage) until it equals the available output current of the generator. Under these stable conditions the terminal voltage is fixed and is said to be corona limited. In general, voltage breakdown across short gaps between smooth electrodes will occur as sparking and across long gaps between points as corona.

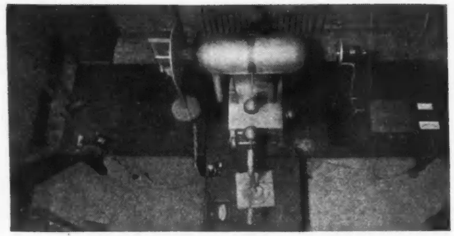

FIG. 5. Top view of assembled electrostatic apparatus.

The breakdown characteristics of a negative point are quite different from those of a positive point. A negative point at high voltage produces a stable "shaving-brush" corona discharge of approximately uniform density, whereas a positive point produces unstable and irregular streamers of dense ionization. If the corona current is measured in the two cases, the negative current will prove to be larger for a given voltage and geometry, and also to remain stable for higher values of voltage. This difference shows itself in generator performance in that a negative terminal is more stable if the voltage is limited by points connected to the terminal, and a positive terminal, if limited by points connected to ground. These differences in performance can be explained qualitatively on the basis of the fact that electrons possess higher mobilities than positive ions. An excellent discussion of the corona from positive and negative points appears in reference 5(a).

5. Apparatus

These electrostatic experiments utilize the following apparatus (Fig. 5):

1. A belt electrostatic generator with a belt 10 in. wide and with 18 in. of column insulation, capable of delivering a current of about 200μa at a voltage of about 175 kv. In spite of its voltage, such a generator is not dangerous to use. A spark from the terminal would be very unlikely to injure

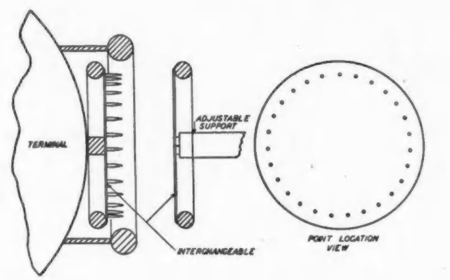

FIG. 6. Corona point arrangement for voltage control.

an operator because of the definite current limitation and the small capacitance between terminal and ground.

2. A set of corona points for voltage control, which can be mounted either on the terminal or on an opposing grounded plate (Fig. 6).

3. A high resistance voltmeter consisting of a spiral composition resistor (IRC type-MV) which has been provided with spun-copper re-entrant electrodes, and a sensitive microammeter. Wire-wound resistors (Shallcross "Taylor") are available for voltages up to 200 kv if the application justifies the expense and if the necessary current (1 ma) can be supplied.

4. A spark gap voltmeter consisting of two 10.2-cm polished spheres located near the terminal with means for varying and measuring the spark gap.

5. An attracted disk voltmeter having a disk 8 in. in diameter, a spring support that permits a motion of 5×10^{-5} mm/dyne and an optical system that multiplies this displacement by a factor of 1000.

6. A generating voltmeter capable of giving full-scale reading on the output meter for electric intensities from 150 to 15,000 v/cm. This voltmeter consists essentially of the following parts: (*a*) an electrostatic alternator in which a double-quadrant shaped rotor 6 in. in diameter driven at 3600 rev/min by a synchronous motor alternately covers and exposes a pair of insulated stator quadrants; (*b*) an amplifier-rectifier unit which gives 1μa d.c. output for 10^{-9} amp a.c. input; (*c*) a gain control to reduce this sensitivity by as much as a factor of 100 in small steps; (*d*) a 100-μa output meter.

7. An individual point-to-plane gap for a detailed study of corona currents in air.

8. A voltage source of about 7 kv (this may be the excitation voltage supply used with the belt generator) together with a 10-megohm precision wire-wound resistor and a meter with a range of 1 ma. These are required in order to obtain a low voltage calibration for the generating voltmeter.

6. Procedure and Results

The generating voltmeter, if properly used, may be expected from general considerations to give the most nearly linear response with voltage of the various voltmeters considered. In practice this proves to be true. It will be taken, therefore, as the voltage standard against which to compare the other instruments.

1. Calibrate the generating voltmeter using the 7-kv power supply and the precision voltmeter.

2. By means of the corona control set the terminal voltage at a number of values. At each setting measure the voltage by means of the generating voltmeter and read also: (*a*) the current in the high resistance voltmeter; (*b*) the gap distance in the spark gap voltmeter, together with the barometric pressure and temperature; (*c*) the displacement in the attracted disk voltmeter.

3. Plot each of these quantities as a function of voltage.

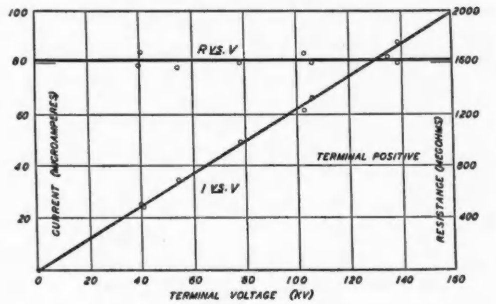

FIG. 7. High resistance voltmeter: current and resistance *vs.* voltage.

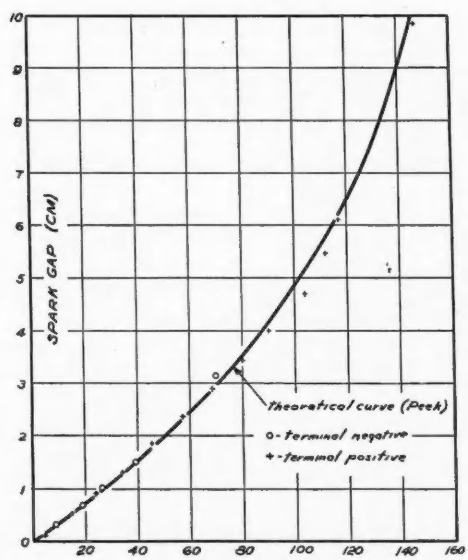

FIG. 8. Spark gap voltmeter: separation *vs.* voltage.

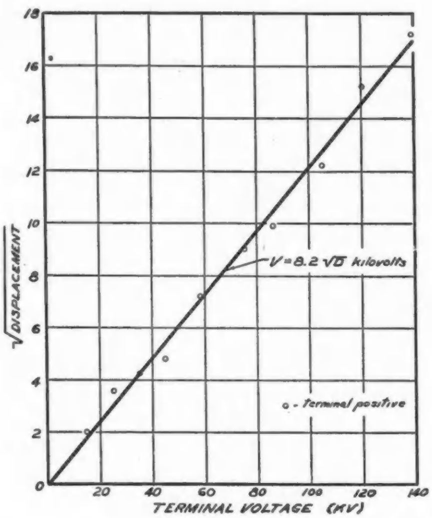

FIG. 9. Attracted disk voltmeter: square root of displacement *vs.* voltage.

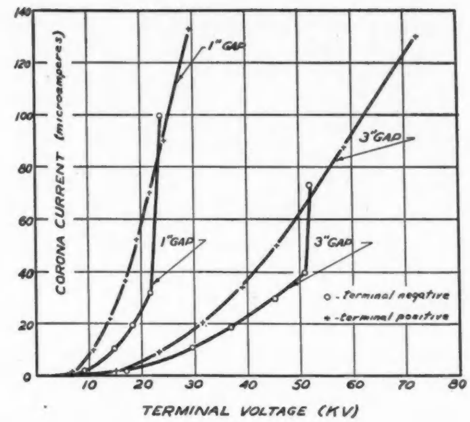

FIG. 10. Corona currents *vs.* voltage; points positive and negative for two gap distances.

4. Calculate and plot as a function of voltage the value of the resistance used in the high resistance voltmeter.

5. Calculate from the general equation given in the discussion the breakdown voltages for the range of spark gap distances used, and plot this curve for comparison with the measured points.

6. Determine the law of displacement for the attracted disk voltmeter and the constant in the equation.

7. For a positive and a negative point and for two values of gap distance, measure the corona current from the point as a function of voltage. Make a note of any instability in performance which may be observed. Plot these data.

The results obtained by three students during a regular three-hour laboratory session are summarized in Figs. 7 to 10. The data are sufficiently accurate to establish the validity of the relations involved and to indicate the possibility of quantitative research in high voltage electrostatics.

We are indebted to Professor R. J. Van de Graaff, Mr. D. L. Northrup and Mr. J. C. Clark of the High-Voltage Laboratory for suggestions and assistance, and to Professor D. C. Stockbarger, who is in charge of the junior physics laboratory and who urged us to develop an electrostatics experiment.

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This incomplete list of references has been chosen for the purpose of illustrating specific points. Many related papers of major importance have not been included.

Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

11. Caricatures of Lectures at the Royal Institution

E. C. Watson
California Institute of Technology, Pasadena, California

THE Royal Institution of Great Britain, founded in 1799 by Count Rumford (see Reproduction 10 in this series) for the purpose of "diffusing the knowledge and facilitating the general introduction of useful mechanical inventions and improvements, and for teaching by courses of philosophical lectures and experiments the application of science to the common purposes of life," has exerted an enormous influence upon the development of science through the original researches of Thomas Young (1773–1829), Humphry Davy (1778–1829), Michael Faraday (1791–1867), John Tyndall (1820–1893), James Dewar (1842–1923), John William Strutt, third Lord Rayleigh (1842–1919), William Bragg (1862–) and others, which have been conducted in its laboratories. An account of the founding, the founders and the early years has been given by Henry Bence Jones,[1] Secretary of the Institution from 1860 to 1873. As he points out, the "usefulness of science to the common purposes of life" gradually ceased to be the primary object; "the school for mechanics, the workshops, the models, the kitchen, and the journals died away and the

[1] H. B. Jones, *The Royal Institution: its founder and its first professors* (London, 1871). See also the *Record of the Royal Institution of Great Britain* for 1939 (Wm. Clowes & Sons, 94 Jermyn St., London, S.W. 1, price, 5 s.); T. Martin, "The professors of the Royal Institution," *Nature* **135**, 813 (1935); F. Cajori, *A history of physics* (Macmillan, 1929), p. 402; A. Wolf, *A history of science, technology, and philosophy in the eighteenth century* (Macmillan, 1939), pp. 42–44.

Plate 1. A Lecture at the Royal Institution (from a caricature by James Gillray).

laboratories, the lectures and the library became the life of the new Institution" and its purpose, "the diffusion and extension of useful knowledge in general."

"Lectures on scientific subjects, to be given in a lecture room with the most up-to-date facilities for experiment and demonstration," were, however, a part of the original scheme. These have continued without interruption to the present day and because of the abilities of the Institution's distinguished line of professors have been just as influential in *diffusing* a knowledge of science and its methods as were the original researches of these men in *adding* to useful knowledge.

Plate 1 is a burlesque, by that incomparable caricaturist, James Gillray (1757–1815), of one of the early lectures. The original is a gayly colored print ($13\frac{1}{2}\times 9\frac{1}{2}$ in.) which was published on May 28, 1802 by H. Humphrey, the publisher of most of Gillray's many caricatures. The lecture room and much of the apparatus are well portrayed and many of the figures in the audience can be identified as the more distinguished members of the Royal Institution. The names of many of these will be recognized even today. Thus the gentleman being experimented upon is the diplomat and politician, John Cox Hippisley (1748–1825), who in 1800 became one of the managers of the Institution. The lecturer is Thomas Garnett[2] (1766–1802), the first professor. The lecture assistant (holding the bellows) is Humphry Davy, soon to become Garnett's successor and the most famous chemist of his age. In the upper right-hand portion of the print (to the left of Davy) Count Rumford is easily recognized; and in the circle beginning with him are Isaac Disraeli (in spectacles), Earl Gower (afterwards Marquis of Stafford), Lord Stanhope (in top-boots and leaning on a stick), Earl Pomfert, Sir Henry Englefield, Miss Lock (afterwards Mrs. Angerstein), Lady Charlotte Denys and her daughter, Mr. Sotheby (with cane), Mr. Denys (in spectacles) with his son, Mr. Tholdal (a German attaché), and others who cannot be identified at this date. On the seat near Lord Stanhope is an open pamphlet entitled, "Hints on the nature of air required for the new French diving-boat." This probably refers to the *Nautilus*, the submarine which Robert Fulton built in 1800 with funds provided by Napoleon and with which he carried out successful tests in the Seine and at Brest.

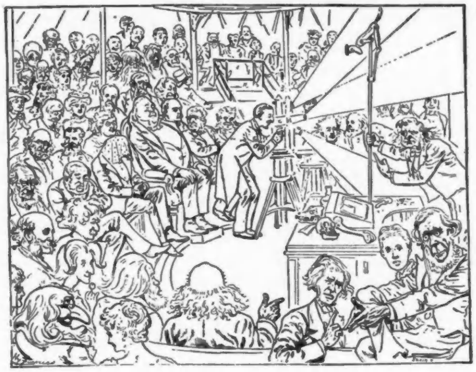

Plate 2. A Friday Evening at the Royal Institution (from a caricature by Harry Furniss).

Plate 2 is from a drawing by Harry Furniss which was reproduced in *Punch*, July 11, 1885. The original hangs in the rooms of the Royal Institution. It caricatures a lecture on evolution by Thomas Huxley. The figure in the lower right-hand corner is of course John Tyndall, but the identification of the rest of the audience will be left to the reader.

These two caricatures, executed more than 80 years apart, tell a remarkable story of the part which the lectures at the Royal Institution played in the life of London during the whole of the nineteenth century.

[2] An account of his life is given by Jones, Reference 1.

Oersted would never have made his great discovery of the action of galvanic currents on magnets had he stopped in his research to consider in what manner they could possibly be turned to practical account; and so we should not now be able to boast of the wonders done by the electric telegraphs. Indeed, no great law in Natural Philosophy has ever been discovered for its practical application, but the instances are innumerable of investigations apparently quite useless in this narrow sense of the word which have led to the most valuable results.—Lord Kelvin.

NOTES AND DISCUSSION

A Theory of e'/m by Deflection Based on Familiar Geometry

THE deflection of the spot in a cathode-ray tube, when a uniform magnetic field perpendicular to the axis of the tube is applied, provides, for lecture demonstration purposes, a very convenient means of determining the charge-to-mass ratio of electrons. In determining the radius of the path from the deflection of the spot, geometry ordinarily is employed, which, although used in several parts of the elementary physics course, is not sufficiently familiar to eliminate the need for derivation. This, by consuming excessive time, may detract from the effectiveness of the demonstration. In the following discussion, the geometry is simplified to the definition of an angle as the ratio of arc to radius; and an important physical principle, not usually emphasized, is clearly shown through the development of the Larmor precessional velocity for electrons.

If an electron is projected with a velocity at right angles to a field of uniform magnetic induction B (gauss), it is acted upon by a force at right angles both to the magnetic field and to its own velocity. This force does no work, but it produces a central acceleration, and the particle moves in a circle of radius r (cm) with a constant speed v (cm/sec). To obtain e'/m (abcoulomb/gm), the charge-to-mass ratio for the electron, we equate the magnetic reaction $Be'v$ (dynes) to the central force $m\omega^2r$ (dynes), where ω is the angular speed (radian/sec) of the electron in its circular path. But $v=\omega r$ and hence

$$\omega = e'B/m. \tag{1}$$

The quantity ω, which is twice the conventional Larmor precessional velocity,[1] is the angular speed of the velocity vector for any electron regardless of its linear speed. The application of this principle is the most direct means for the solution of many problems.[2]

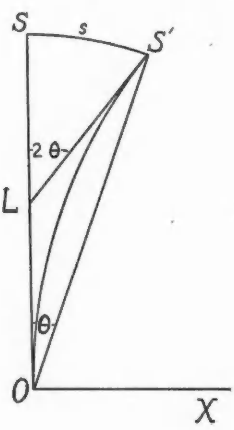

FIG. 1. Theory of e'/m by deflection.

In Fig. 1 the electrons leaving a gun at O will, in the absence of a magnetic field, travel along the straight path OS and strike the fluorescent screen at S. The curved line OS' is assumed to represent the corresponding trajectory in the magnetic field. It will be seen that the velocity of an electron is rotated through an angle 2θ, where θ may be computed as the ratio of the arc SS' to the path length L. Hence $2\theta=\omega t$, where t is the time for the transit of an electron between O and S'. This time t is given by L/v, assuming that the length of the path OS' does not differ appreciably from the length L. Thus

$$2\theta = \omega L/v. \tag{2}$$

Assuming energy to be conserved, we may express v in terms of the potential difference V' (abvolts) through which the electron has fallen as

$$v = (2e'V'/m)^{\frac{1}{2}}. \tag{3}$$

Substituting in Eq. (2) this value of v and the value of ω given by Eq. (1), we obtain $e'/m = 8\theta^2V'/B^2L^2$, or, replacing θ by s/L, $e'/m = 8s^2V'/B^2L^4$.

In this discussion the geometrical quantities are those most directly computed from the measurements, and the major portion of the time is devoted to important physical principles that should be emphasized because of their wide applicability.

PAUL L. COPELAND

Armour Institute of Technology,
Chicago, Illinois.

[1] The factor of two comes from the fact that the velocity vector rotates with twice the angular speed of the displacement vector as shown in Fig. 1 and the accompanying discussion.

[2] In this connection we may note that $\omega/2\pi$ is the frequency for the oscillator driving a cyclotron.

A.C. Voltage Supply for Spectrum Tubes

THE apparatus described in this note is a complete unit used for running spectrum tubes. It is quiet, compact and economical, and can be used safely by students in undergraduate laboratories. The apparatus consists of a high voltage source and several mounted tubes. We use hydrogen, helium, neon, argon and nitrogen tubes, each in its own mount. The source is a 120/4000-v neon sign transformer. Commercial neon sign transformers have the advantage of availability and low price, but the disadvantage that those which have a high enough secondary voltage will overheat the spectrum tubes. To prevent this overheating we put a 400-ohm, 50-w resistor in series with the primary. This limits the power output without appreciably affecting the peak secondary voltage. The principal features of the design are the methods chosen to protect the students against shock and to provide for the protection and interchangeability of the tubes.

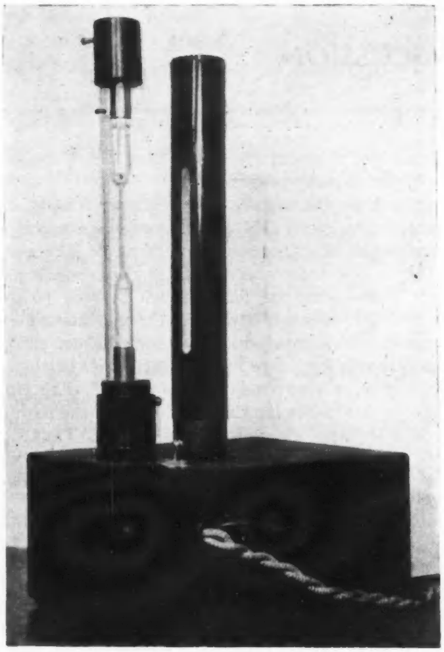

FIG. 1. Apparatus with Bakelite tube cover removed.

Shock protection is provided by mounting the transformer and resistor in a totally enclosed and grounded metal chassis (Fig. 1). The high voltage leads terminate at a 4-prong radio tube socket. The tube and all connections to it are entirely enclosed in a Bakelite case.

The tube mount (Fig. 2) consists of a $1\frac{1}{4}$-in. O.D. Bakelite tube *A* in which there have been cut three $\frac{1}{4}$-in. longitudinal viewing slots 120° apart. This tube fits into a 4-prong radio tube base *G*. Some height adjustment is possible by sliding the spectrum tube in the holder, and it is also possible to change the height in steps of 1 in. by inserting adapters between the radio tube base and socket. Two Bakelite plugs *B* are fitted into the ends of the Bakelite tube. These plugs have holes in them which receive the ends of the spectrum tube. The spectrum tube is then held in position by the phosphor bronze springs *C*, which also provide electrical connections to the tube. The lower connection is made by means of a fine copper wire connected directly to one prong of the tube base. The upper connection is completed through two pieces of No. 14 B & S gauge wire *D*, separated by a spring *F*. Both wires and the

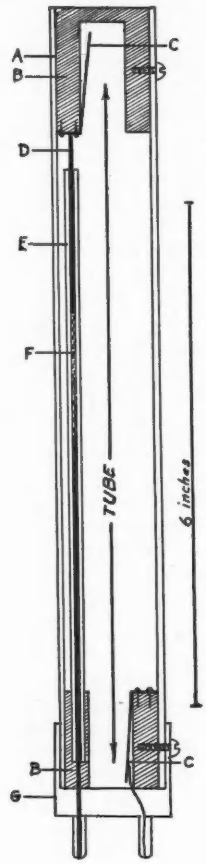

FIG. 2. Details of tube mount.

TABLE I. *Materials and costs.*

ITEM	VENDOR	SPECS. OR CAT. NO.	UNIT PRICE
Transformer	Jefferson Transformer Co.	720–371	$2.33
Chassis	Karp Metal Products Co.		1.00
Paint	Sherwin-Williams Paint Co.	Kem-finish dull black	2.50/gal
D.P.S.T. toggle switch	Radio Wire Television Inc.[1]	K12817	0.26
Resistor, 400 ohms	Radio Wire Television Inc.[1]	K20773	0.66
Tube socket	Radio Wire Television Inc.[1]	K13072	0.18
Bakelite tube *A*	Formica Insulation Co.	XPB $1\frac{1}{4}$ in. O.D., $\frac{1}{8}$ in. wall	0.50/ft
Plugs *B*	Formica Insulation Co.	$1\frac{1}{8}$ in. grade XX Bakelite rod	0.35/ft
Spectrum tubes	Tubelite Engineering Co.		2.50
Radio tube bases	Radio Wire Television Inc.[1]		0.15
Adapter	Radio Wire Television Inc.[1]	K11562, K11569	0.24/set

[1] Radio Wire Television Inc. was formerly Wholesale Radio Service Co.

spring are housed in a small glass tube E which is supported in a hole in the lower plug. To replace a defective tube only the upper plug B need be removed.

If ground connections are not available in the laboratory, they can usually be provided by attaching a binding post to the electrical conduit; that is, to the outlet cover plate if it is made of metal, or by drilling through the plate to the outlet box itself if the plate is of some insulating material. It is important that these ground connections be tested, and it is advisable that all ground connections on the apparatus be soldered. All ground connections at the tables should be removable only with the use of tools.

Twelve high voltage sources and 48 tube mounts were constructed in our department shop and are now in use in the optics laboratory. The labor cost for one source is approximately the same as for one tube mount, each requiring about four man-hours when made in reasonable quantities. A list of the materials and costs is given in Table I. The cost, including tubes but exclusive of labor, is $7.73.

E. H. Green
K. H. Fried
W. H. Mais

Brooklyn College,
Brooklyn, New York.

All Conversion Factors Are Unity

We teachers of physics take pride in the idea that our chosen subject is the most precise of all subjects. In action, however, we are often careless and inaccurate. In the handling of units, for instance, we often do not mean that which we actually write or say. We have other ways of carelessness, but it is the transformation, or conversion, factor which enters when we transform or convert the expression of a physical quantity from one unit to another that interests us here. My text is, "All conversion factors are unity." They are nothing more nor less than the numeric "one" written in each instance in a manner suitable for the occasion.

Consider the problem of finding the acceleration, strictly its scalar value, for the case where an automobile, moving with a speed of 45 mi/hr, is brought to rest at a constant rate in 5.0 sec. We write, at least many of us do,

$$a=-45/5.0=-9 \text{ mi/hr/sec},$$

little realizing that we have offended at least twice. Others will say you cannot express the acceleration that way, and there is "more truth than poetry" in that statement.

But, assuming a more approved form for the acceleration, let us see what may be done, using conversion factors involving the desired units. We have, making use of the canceling privilege,

$$a=-9.0\frac{\text{mi}}{\text{hr sec}}\times\frac{5280 \text{ ft}}{1 \text{ mi}}\times\frac{1 \text{ hr}}{3600 \text{ sec}}$$
$$=-9.0\times\frac{5280}{3600}\text{ ft/sec}^2=-13.2 \text{ ft/sec}^2, \quad (1)$$

of which the factors *5280 ft/1 mi* and *1 hr/3600 sec* are merely convenient forms for the numeric "one." That such is the case is evident to us all. Students, however, need to be shown that

$$5280 \text{ ft}=1 \text{ mi},$$
$$5280 \text{ ft}/1 \text{ mi}=1 \text{ mi}/1 \text{ mi}=1.$$

Having convinced one's students that conversion factors are unity for all cases, it is also easy to convince them that the acceleration 9.0 mi/(hr sec) is exactly the same acceleration as 13.2 ft/sec^2. The question is: Is it not better to follow the method indicated and to emphasize that the conversion factors are unity than, after having obtained 9.0 mi/(hr sec), to say "now, in order to get the acceleration in ft/sec^2, we must multiply what we have obtained by 5280 and divide by 3600"? Whether or not we individually speak thus of multiplying and dividing, there are many who say exactly that, and our physics text books will serve as proof of the statement.

Some will say, what is the difference? They forget for the moment their pride in the fact that physics can be the most precise of all sciences. Aside from that, it emphasizes that a change in velocity divided by the corresponding change in time yields an acceleration, regardless of the units used in expressing the change in velocity or the corresponding time interval. Further, the process of transferring from one unit to another is almost "student proof."

As another illustration, consider converting the universal gravitational constant from the form involving cgs units to that involving engineering units. Introducing needed conversion factors for canceling undesired units and introducing those desired, we have

$$G=6.66\times10^{-8}\frac{\text{dyne cm}^2}{\text{gm}^2}\times\frac{1 \text{ gm cm/sec}^2}{1 \text{ dyne}}\times\frac{454 \text{ gm}}{1.00 \text{ pd}}$$
$$\times\frac{1.00 \text{ ft}^3}{(30.5 \text{ cm})^3}\times\frac{32.2 \text{ pd ft/sec}^2}{1.00 \text{ lb}}$$
$$=6.66\times10^{-8}\times\frac{454\times32.2}{30.5^3}\;\frac{\text{ft}^4}{\text{lb sec}^4}. \quad (2)$$

The procedure is simple and direct. With its aid, most if not all conversions are readily made; but, without some such aid, many advanced and graduate students in mathematics, engineering and physics, whom the writer has had in his classes, have been stopped by the problem just considered.

There is another logical, much used method of converting compound units, namely, that of substituting for each component unit to be converted its equal in the system desired. Which method is preferable, is a question that individual instructors will answer differently. The writer feels, however, that the method involving conversion factors has in its favor the facts that it presents, as a part of the solution of a problem, all the details of conversion in such manner that they may be easily checked, and emphasizes that units, like numerics, are subject to the ordinary laws of arithmetic and should be so handled.

In a course given by the writer, entitled "Basic Concepts" the concept that *all conversion factors are unity* is presented early. The expressions of satisfaction and the show of confidence that has resulted has left no doubt in the writer's mind as to its value.

A. G. Worthing

University of Pittsburgh,
Pittsburgh, Pennsylvania.

Summary Courses and Symposiums

For information concerning summer courses at Cornell University, see the April issue, page 140.

Armour Institute of Technology

July 15 to August 10

Tensor Analysis of Electrical and Mechanical Engineering Problems. G. Kron, General Electric Company.

Electrical Radiation and Radiating Systems for Radio Communication. W. L. Everitt, Ohio State University.

August 12 to September 7

Physics of Electron Tubes. P. L. Copeland.

Columbia University

J. C. Slater, of Massachusetts Institute of Technology, will give courses on *Theory of Electricity and Magnetism* and *Structure of Matter;* the latter course will deal with thermodynamics and statistical mechanics applied chiefly to the properties and structure of solids.

Other graduate courses: *Analytical Mechanics, Statics,* R. von Nardroff; *Analytical Mechanics, Kinetics,* R. von Nardroff; *Physical Laboratory,* selected experiments in the newer developments of physics or requiring specialized equipment, designed particularly for teachers, G. N. Glasoe and C. G. Stone.

Massachusetts Institute of Technology

Spectroscopy conference. July 15–17. Morning and afternoon discussions on spectroscopic analysis of materials and on other applications of spectroscopy to biology, medicine, chemistry, metallurgy, mineralogy, and to industrial and engineering problems. Attendance limited to 200; no fee.

Courses in practical spectroscopy. June 10–July 20. Lecture and laboratory courses on spectroscopic analysis of materials by study of their emission and absorption spectra, and on the general application of the spectroscope and its auxiliary apparatus to problems of science and industry.

Spectroscopic research. June 1–August 1. Qualified spectroscopists and also students from other institutions who are engaged in thesis work involving spectroscopy may arrange to use the facilities of the laboratory. Unusual equipment is available in the fields of intensity and wave-length measurements, vacuum spectroscopy, analysis of spectra and spectroscopy at high resolution.

Conference on friction and surface finish. June 5–7. Problems concerning boundary lubrication, the critical and illusive phenomenon of friction which occurs just before failure.

Conference on the design and use of the differential analyzer. July 8–12.

Conference on powder metallurgy. August 29–31.

Stanford University

The graduate program for the summer quarter, June 20 to August 31, includes courses on *Nuclear Physics,* by H. A. Bethe, of Cornell University, and on *Theory of Collisions,* by F. Bloch. The regular research activities of the department will be carried on in the fields of x-rays, neutrons, atmospheric electricity, conduction of electricity in gases and high frequency radio.

University of Chicago

During the summer quarter, June 17 to August 23, special weekly seminars will be held on problems in nuclear physics, cosmic rays and spectroscopy. The advanced courses to be offered include:

Structure of Molecules and of Liquids from the Point of View of Interference Phenomena. P. Debye.

Theoretical Aspect of Cosmic Rays (First term only). E. Fermi.

Cosmic Rays and Allied Problems (First term only). B. Rossi.

X-Rays and Electrons. A. H. Compton.

Classical Mechanics. W. H. Zachariasen.

Thermodynamics. A. J. Dempster.

Electrodynamics I. F. C. Hoyt.

Statistical Theories of Thermodynamics. C. Eckart.

Line Spectra and Atomic Structure. C. Eckart.

Band Spectra and Molecular Structure (Diatomic Molecules). R. S. Mulliken.

University of Michigan

The annual symposium in theoretical physics will be held from June 24 to August 16. Nuclear physics will be stressed, but other subjects will be presented, particularly in the bi-weekly seminars and special lectures. Series of special lectures have been arranged as follows:

Recent developments in the theory of atomic nuclei. E. P. Wigner, *Princeton University.* Throughout session.

Band spectra. D. M. Dennison, *University of Michigan.* Throughout session.

Theoretical aspects of cosmic rays. G. E. Uhlenbeck, *University Michigan.* Throughout session.

Recent field theories, especially in connection with the theory of the meson. R. Serber, *University of Illinois.* June 24–July 7.

Special topics in the theories of radiation and of β-radioactivity. W. H. Furry, *Harvard University.* July 1–14.

Low temperature physics; properties of liquid helium; supraconductivity. F. W. London, *Duke University.* July 7–28.

Recent experimental results in cosmic rays. B. Rossi, *University of Chicago.* July 28–August 5.

The following advanced courses will be offered: *Electrical Measurements,* R. A. Sawyer; *Electron Tubes,* N. H. Williams; *Mechanics of Solids,* G. A. Lindsay; *Heat and Heat Laboratory,* J. M. Cork; *X-Rays,* G. A. Lindsay; *Atomic and Molecular Structure,* E. F. Barker; *Nuclear Physics and Nuclear Physics Laboratory,* H. R. Crane; *Thermodynamics,* G. E. Uhlenbeck; *Quantum Theory and Atomic Structure,* D. M. Dennison; *Conduction of Electricity Through Gases,* O. S. Duffendack; *Band Spectra,* D. M. Dennison.

Students desiring experience in industrial research as a prelude to a later professional career in industry will find summer courses and practical work offered in the fields of *sound* and *noise photographic spectroscopy,* and *infra-red spectroscopy.* No one should come for practical work in any of these fields without first receiving permission.

University of Minnesota

First Session

Analytical Mechanics, E. L. Hill; *Physics of Vacuum Tubes*, A. O. C. Nier; *Demonstrations in Physics for High School Classes*, J. W. Buchta.

Second Session

Principles of Photography, J. Valasek; *History of Physics*, L. H. Rumbaugh; *Kinetic Theory of Gases*, J. Bardeen; *Experimental Optics*, J. Valasek.

University of Iowa

The colloquium of college teachers, June 13 to 15, will include the following lectures:

Recent research and the teacher. E. P. Tyndall.
Opportunity in the laboratory. Round table discussion led by J. A. Eldridge.
Laboratory demonstrations. P. L. Copeland, C. A. Culver, R. L. Dolecek, R. E. Harris, A. G. Hoyem, R. A. Nelson, C. R. Smith, E. R. Wightman, R. C. Wyckoff.
Facing the facts. L. M. Heil.
Physics as a cultural subject: results with demonstration lectures. W. S. Webb.
What material may be eliminated in the general college course. O. M. Stewart.
Book experiences. G. W. Fox, J. W. Hake, J. Harty, W. H. Kadesch R. R. Palmer, T. Smith.
Gasoline for the physicist. P. D. Foote.
The world's last frontier. T. C. Poulter.
Experiences in acoustic and radio engineering. H. S. Knowles.
Physics and the violin. A. M. Small.

Requests for reservations in the University dormitories should be sent early to G. W. Stewart.

University of Pennsylvania

Advanced Laboratory Work. Experiments in optics, electricity and atomic physics, selected to meet the needs of individual students. 3 hr daily. L. N. Ridenour and P. L. Bayley.

Classical Physical Methods. Analyses of advanced undergraduate problems and a comprehensive study of methods of solutions, designed to increase the proficiency of teachers and others through a better understanding of fundamental principles. 1 hr daily. I. C. Cornog.

Modern Physical Theories. Modern quantum theory, from its experimental beginnings to the development of Dirac's relativistically invariant formulation, with emphasis on the logical aspects of the developments rather than on their mathematical minutiae. 1 hr daily. L. N. Ridenour.

These graduate courses are planned mainly for candidates for the Master's degree and for students in the School of Education.

University of Pittsburgh

Advanced courses will be offered in the summer session, June 17 to July 26, as follows:

Introductory Electron Theory of Metals. L. A. DuBridge, *University of Rochester.*
Nuclear Physics, L. A. DuBridge.
Electrical Properties of Metals. W. V. Houston, *California Institute of Technology.*
Recent Experiments with Metals. R. P. Johnson, *General Electric Company.*
Structure of Glasses. B. E. Warren, *Massachusetts Institute of Technology.*
X-Ray Diffraction. B. E. Warren.
Seminar in Nuclear Physics. E. U. Condon, *Westinghouse Research Laboratory.*

University of Wisconsin

The following courses are among those that will be offered during the six-weeks session, from June 24 to August 2, and the eight-weeks session, from June 24 to August 16: *Photography*, J. E. Mack; *Physical Optics*, L. R. Ingersoll; *Kinetic Theory of Matter*, L. R. Ingersoll; *Radiation*, L. R. Ingersoll; *Heat* (6 and 8 wks), J. E. Mack; *Atomic Structure* (6 and 8 wks), J. E. Mack.

Clement Moran, 1884–1940

ASSOCIATE Professor Clement Moran of the University of New Hampshire was stricken with a heart attack while attending a meeting of the New England section of the American Physical Society on January 26, at the Massachusetts Institute of Technology, and died a few minutes later in the infirmary of that institution.

Professor Moran was born on October 30, 1884, in Black Run, Ohio. He was graduated from Defiance College with the degree of Bachelor of Arts. His first teaching post was at Starkey Seminary, Lakemont, New York. He was appointed an instructor at New Hampshire College in 1914, an Assistant Professor in 1921 and Associate Professor in 1928. During the summers of 1917–1919 he was a member of the staff in the standardizing laboratory of the General Electric Company. His second degree was taken at Cornell University, with a thesis on "Effect of the size of ions on the structural temperature of water."

Coming to the University of New Hampshire as a young teacher, he built himself into the organization and spirit of the institution. He was at the same time active in town affairs and campus societies, serving in offices such as town auditor, treasurer of the Red Cross and treasurer of the Society of Phi Kappa Phi. His hobby was photography, which he developed into a useful art for the university extension and for demonstration work. Because of his eminent fairness and his willingness to work long hours with his students, he was very popular with them. The development of the electrical measurements laboratory and, with the late Dr. Willard J. Fisher, that of the sophomore laboratory in engineering physics are memorials to his marked ability in laboratory work.

Professor Moran was married in 1918 to Mary E. Duncan of Hamilton, Ohio, who with her two sons, Harold Duncan Moran and Charles Vernon Moran, survive him.

Horace L. Howes

Albert DeForest Palmer, 1869–1940

THE faculty of Brown University record with sorrow the death on January 13, 1940, of Emeritus Professor Albert DeForest Palmer in Pasadena, California.

Born in 1869 in Tewksbury, Massachusetts, Professor Palmer received his Ph.B. degree at Brown University in the class of 1891. After pursuing graduate work in physics

at the Johns Hopkins University from 1891 to 1893 he returned to Brown as Instructor. Here he also completed his graduate work and obtained the doctor's degree in 1895. He was made Associate Professor in 1896 and held this post for 38 years until his retirement in 1934. From 1926–1934, Professor Palmer served as Chairman of the Department. All told, his active association with Brown University as student and teacher covered a span of some 45 years.

In spite of the rather meager equipment at his disposal Professor Palmer carried out a considerable amount of experimental investigation of very high order and became a research physicist of national reputation. His work was not spectacular and he made no effort to achieve the headlines. He was slow and painstaking and was never satisfied until he could attain the ultimate in preciseness. Indeed, precision measurement was his forte, and one cannot read his published papers without realizing the care he took in arranging his apparatus for maximum efficiency and the delight he experienced in the fine points of physical measurement. The breadth of his work is indicated by the fact that his research covered such diverse fields as the dielectric constant of water, the measurement of high pressures, the wave-length of spectral lines, electrical resistance measurement and the properties of polarized light. Evaluated in number of papers published, Professor Palmer's research output was small; nevertheless, each paper was not a mere catalog of results obtained but showed thorough mastery of the relevant material. He was always reluctant to publish anything until he thoroughly understood the significance of his findings. His interest in precision measurement finally led him to write *The Theory of Measurements* (1912); it became a standard textbook in this field and its utility has been emphasized by many.

Though a clear lecturer, Professor Palmer's ability as a teacher was most clearly evident in the small discussion group or laboratory class where he could meet each student individually. No student who ever consulted him about a problem came away without being impressed by his fundamental grasp of the subject, his inexhaustible patience and good nature and his essential kindliness.

Though plagued intermittently by ill health for the last ten years of his active teaching career, Professor Palmer kept at his work with stubborn but cheerful endurance. It was the hope of his many friends that after his retirement and subsequent establishment of a home in California he would enjoy sufficiently good health to enable him to continue experimental investigations in the laboratory he built for himself there. For a few years this hope was realized, but unfortunately his strength was ultimately not equal to the strain placed upon it.

R. B. LINDSAY

Appointment Service

REPRESENTATIVES of departments or of institutions having vacancies are urged to write to the Editor, Columbia University, for additional information concerning the physicists whose announcements appear here or in previous issues. *The existence of a vacancy will not be divulged to anyone without the permission of the institution concerned.*

POSITIONS WANTED

33. M.S., experimental physics, coupled with thorough background of courses in professional education. Has taught physics and mathematics for 3 yrs in large high school. Desires position as instructor in high school physics in a university or college experimental or training school.

34. Ph.D., M.S., Penn State, Age 38, married. 13 yrs teaching experience in colleges and universities; 3 yrs head of department in small college; industrial research experience. Interested in teaching, research and administrative work in a small college.

35. Ph.D., Purdue; M.A., British Columbia. Age 27, married. Experience: 5 yrs university teaching; 2 yrs secondary school teaching; 5 yrs research in analysis of liquids by x-rays. Interested in teaching and research.

36. Ph.D., Pennsylvania '37; M.S., A.B., West Virginia. Age 30, married. Has taught 4 yrs in small liberal arts college of good standing. Interested in a position of greater responsibility and opportunity.

37. Ph.D., engineering physics, mathematics and physical chemistry, Illinois; A.B., education. Age 37, married. University, teachers college, junior college and high school experience; 1 yr editorial work. Especially qualified for survey courses.

38. M.S., physics, Lehigh. Age 35, married. 10 yrs teaching college and university, particularly laboratory instruction. Interested in college teaching, or in industrial production and developmental work in precision instruments.

39. Ph.D., physics, large Eastern university. Age 45, married. 16 yrs university and college teaching experience, including 9 yrs head of small liberal arts college. Research in optics and spectroscopy. Desires opportunity for research in teaching or industrial fields.

Activities of Association Chapters

CHICAGO CHAPTER

ON April 13 the Chicago chapter held a luncheon at the Lawson Y. M. C. A. and an afternoon session at which Professor R. E. Harris presided. P. A. Constantinides, of Wright Junior College, described a simplified method for demonstrating the Stefan-Boltzmann law. L. I. Bochstahler, of Northwestern University, reported on the results of a questionary sent to physics teachers in Illinois colleges regarding the requirements for students majoring in physics. C. A. Benz, of Hammond (Indiana) High School, discussed the means used and the results obtained in a class in remedial arithmetic which he has conducted. The session closed with a discussion of teaching objectives.

KENTUCKY CHAPTER

The Kentucky chapter held a luncheon meeting at Centre College on January 20. Plans were made for the next two meetings. A committee was appointed to devise methods for providing practical help to high school teachers of physics.

On April 19 the Chapter met in the Derby Room of the Brown Hotel, Louisville, in conjunction with the annual meeting of the Kentucky Education Association. The program, designed to have a popular appeal to teachers in all fields, included addresses by Professor W. S. Webb, on "Seeing sound," and by Professor J. G. Black, on "A direct measurement of human horsepower."

RECENT PUBLICATIONS AND TEACHING AIDS

Advanced Physics

Principles of Quantum Mechanics. ALFRED LANDÉ, Professor of Physics, Ohio State University. 129 p., 15 fig., 13×21 cm. *Cambridge Univ. Press* and *Macmillan*, $2.25. Intended for readers who are familiar with the elements of the field, this book critically re-examines the concepts of quantum mechanics and develops its principles on the basis of a few standard observations. The mutual dependence of the principles and the necessity for consistency in their interpretation are stressed. The main emphasis is on the perfect complimentarity of waves and corpuscles, each statement made in wave language being contrasted with the complimentary statement expressed in corpuscular terms. One third of the book deals with the elementary theory of observation; the remainder, with the principles of uncertainty, interference correspondence and invariance.

Internal Constitution of the Earth. Ed. by BENO GUTENBERG, Professor of Geophysics, California Institute of Technology. 423 p., 69 tables, 27 fig., 27×17 cm. *McGraw-Hill*, $5.

Terrestrial Magnetism and Electricity. Ed. by J. A. FLEMING, Director, Department of Terrestrial Magnetism, Carnegie Institution of Washington. 816 p., many diagrams and photographs, 25×17 cm. *McGraw-Hill*, $8.

Because it has become increasingly clear that meteorology, terrestrial magnetism, geodesy, oceanography and many other related subjects are best regarded as branches of geophysics—namely, the study of the earth by physical methods—and because the increasing interest in geophysics in this country has not been matched by the publication in English of systematic treatises on the field, the National Research Council, beginning in 1926, formed various committees to prepare a series of related monographs on the *Physics of the Earth*. A main purpose of the series is to give scientists who are not specialists in geophysics a knowledge of the present status and outstanding problems of the field. The first six monographs, dealing with volcanology, the figure of the earth, meteorology, the age of the earth, oceanography and seismology, respectively, were published as bulletins of the National Research Council and may be obtained from its publication office in Washington, D. C. The series is continued with the present books.

Internal Constitution of the Earth has for its major topics the origin of the solar system, relevant facts and inferences from field geology, elastic properties of materials of the earth's crust, temperatures in the earth's crust, and the interior temperature and cooling of the earth. The contributors to the 16 chapters are L. H. ADAMS, REGINALD A. DALY, B. GUTENBERG, HAROLD JEFFREYS, WALTER D. LAMBERT, JAMES B. MACELWANE, C. F. RICHTER, C. E. VAN ORSTRAND and H. S. WASHINGTON. Each chapter is provided with a bibliography.

Terrestrial Magnetism and Electricity provides a comprehensive review of present knowledge of the earth's magnetic and electrical phenomena. The 13 chapters deal with the earth's magnetism and magnetic surveys, magnetic instruments, magnetic prospecting, atmospheric electricity, instruments for studies of atmospheric electricity, earth-currents, causes of the earth's magnetism and its changes, radio exploration of the outer atmosphere, the F_2-region, the aurora polaris, clouds and their electrical effects, etc. One chapter of 99 pages is devoted to bibliographical notes, a classified list of 1523 selected references, and lists of serial publications, periodicals, societies and other organizations devoted to the field. The contributors to the volume are: J. BARTELS, L. V. BERKNER, J. A. FLEMING, O. H. GISH, H. D. HARRADON, C. A. HEILAND, E. O. HULBURT, H. F. JOHNSTON, H. E. MCCOMB, A. G. MCNISH, W. J. ROONEY, B. F. J. SCHONLAND, O. W. TORRESON and L. VEGARD.

Photography

The Little Technical Library: Photographic Series. 10 vol., 1110 p., illustrated, 12×17 cm. *Ziff-Davis*. 50 cts/vol. The titles in this series are: (1) W. E. Dobbs and C. A. Savage, *Your camera and how it works;* (2) A. and D. Bernsohn, *Developing, printing and enlarging;* (3) W. B. Shank, *Filters and their uses;* (4) K. Heilbron, *Composition for the amateur;* (5) H. C. McKay, *Movie making for the beginner;* (6) I. Dmitri, *Color in photography;* (7) H. Lambert, *Child photography;* (8) M. Seymour and S. Simons, *Home portraiture and make-up;* (9) *Tricks for camera owners* (selected from the magazine, *Popular Photography*); (10) F. Fenner, ed., *A glossary for photography.* These books contain many practical suggestions of value to the person already acquainted with photographic theory as well as to the novice for whom they were written. Several of the volumes, notably 1, 3, 4, 5, 6 and 8, contain better expositions of their subjects than are to be found in the more elementary college photography texts that have been published recently, though the very informal style of some of the authors may be found objectionable by college students. —W. W.

History and Biography

Rutherford—Being the Life and Letters of the Rt Hon. Lord Rutherford, O. M. A. S. EVE, formerly Macdonald Professor of Physics, McGill University. 467 p., 18 plates, 6 fig., 17×25 cm. *Macmillan*, $5. This is the authorized life and letters of Lord Rutherford, "the Newton of the atom" and the outstanding experimental physicist of the age. It is written in the language and in the spirit of physics by a former colleague who had free access to the lectures, books, papers, and private notes and corre-

spondence left behind by Rutherford and carefully preserved by his widow and co-workers. The book should be in the library of every department of physics.

The Photismi De Lumine of Maurolycus. Translated by Henry Crew, Professor Emeritus of Physics, Northwestern University. 151 p., 5 plates, 69 fig., 15×22 cm. *Macmillan*, $3. The *Photismi de lumine* of Franciscus Maurolycus (1494–1575) was first published in Latin in 1575, although a major portion of it had been written by 1554. It "represents the best thought in optics some four hundred years ago when spectacles had come into rather wide use but were imperfectly understood, when the structure of the eye and the functions of its various parts were little known and when color was completely unexplained." As the text in any of its several editions is now quite rare and the ability to read Latin even rarer among physicists, this annotated translation is very welcome indeed. It is executed with the same scholarship and care that characterized Professor Crew's earlier translation of Galileo's *Two New Sciences* (Macmillan, 1914), a work which has filled a real need, and one may with confidence predict that the present translation will also serve a very useful purpose. Valuable features added by the translator are footnotes, supplementing those of Father Clavius, and an interesting historical introduction which concludes with an eleven-page biography of Maurolycus. There is also a short preface, a table of contents and a brief index. The frontispiece reproduces an excellent portrait of Maurolycus which is carefully authenticated, a practice which cannot be too strongly commended. All in all this is a book which should find its place on the shelves of every physics library.—E. C. W.

Miscellaneous Books

The Serial Universe. J. W. Dunne, fellow Royal Aeronautical Society. 240 p., 30 fig., 27 tables. *Macmillan*, $2. A person observing a phenomenon is conscious of two things: observing is being done and it is himself who is doing it. In an earlier book, entitled *An Experiment with Time* (Macmillan, ed. 4, $2.75), Dunne considers this *consciousness* equivalent to *observation*, by some observer No. 2, of the mental processes of observer No. 1. The assumption of this equivalence seems to be one of the weakest points in his theory. Once granted, however, one can go to observer No. 3, etc. In this sense an observer is a "serial" observer. Time is also "serial." The unfolding of successive events may be considered due to our traveling forward along the time dimension. But it takes time—time No. 2—to travel along the dimension of time No. 1, etc. To the observer No. 2 in his time No. 2 the events of the past, present and future of our ordinary time No. 1 are ever present; thus, when he is asleep and therefore free from the domination of observer No. 1, he can observe the future as easily as the past. Dunne cites numerous examples of dreams of his own and of others that have come true. In his more recent book, *The Serial Universe*, Dunne reconsiders the laws of physics in the light of the concept of serial time. Finite velocity of light, relativity and quantum mechanics he finds to be necessary consequences of his serialism. The universe is determinate and classical; the Heisenberg principle of uncertainty is traced to the uncertainty about the nth observer by the $n+1$th observer. Finally he deduces personal immortality; we die only in the time No. 1; in time No. 2 we persist forever.

The methods of proof are rather interesting even if not always convincing. Dunne illustrates his ideas graphically; then, by a sort of geometric extrapolation, he squeezes out of his graphs information beyond that which the graphs were set out to represent. Such methods can be justified only by their results; but none of Dunne's contentions is amenable to experimental test, which means that they lie beyond the realm of natural science as we understand it today.—L. N.

Modern Methods and Materials for Teaching Science. Elwood D. Heiss, Science Department, State Teachers College, East Stroudsburg, Pa.; Ellsworth S. Obourn, Science Department, John Burroughs School, Clayton, Mo.; and C. Wesley Hoffman, Science Department, Blair Academy, Blairstown, N. J. 361 p., 27 fig., 5 tables, 14×22 cm. *Macmillan*, $2.50. Designed to serve as a textbook for a college course in methods of teaching the several school sciences, and as a source of useful information for science teachers, this book is divided into three sections: (1) principles of science teaching, including the psychological basis, objectives and modern laboratory, demonstration and testing practices; (2) visual and other teaching aids; (3) sources of useful charts, exhibits, pamphlets and motion pictures.

Research and Statistical Methodology Books and Reviews, 1933–1938. Oscar Krisen Buros, Editor. 105 p., 15×23 cm. *Rutgers Univ. Press*, $1.25. This volume lists 209 books published in 1933–1938 on research and statistical methodology in various fields of knowledge. Under each title is given complete bibliographic information and evaluating statements excerpted from critical reviews of the book published in American and British learned journals.

Charts and Posters

Tools. Set of charts, 46×46 cm. *Henry Disston & Co.* (Philadelphia, Pa.), gratis. Shows handsaws, files and hacksaws.

First Aid. 41×64 cm. *Fisher Scientific Co.* (Pittsburgh, Pa.), gratis. Laboratory emergency chart.

Pamphlets

Booklets on Buying Household Equipment. *Good Housekeeping Bulletin Service* (57th St. at 8th Ave., New York), 3 cts each. Each booklet concisely describes the essential differences in types, finishes, etc. that affect individual choices of equipment. Some of the titles are: electric range, gas range, automatic refrigerator, ice refrigerator, electric toaster, electric ironer, electric iron, electric waffle iron, washing machine, vacuum cleaner.

Periodicals

Tin and Its Uses. *International Tin Research & Development Council* (Fraser Road, Greenford, Middlesex, Eng.), gratis. A quarterly review of technical information relating to the production and uses of tin, its alloys and compounds.

DIGEST OF PERIODICAL LITERATURE

Apparatus and Demonstrations

Heat transfer through a gas. R. G. Mitton; *Sch. Sci. Rev.* **21**, 883–885 (1939). About 50 cm of No. 26 iron wire is wound in the form of a small helix and the ends of the wire are silver-soldered to short lengths of sealing-in wire which are brought out through seals in the side of a glass bulb. The bulb is connected by tubing to a mercury manometer and a vacuum pump. The wire is chosen so that its resistance at room temperature is about 0.6 ohm. It is heated by a current until its resistance R is exactly 1 ohm, in which condition the reading of a voltmeter connected across its terminals is numerically the same as that of an ammeter in series with it. This value of R and, hence, of the temperature of the wire is maintained by adjusting the current while the pressure of the gas in the bulb is varied by means of the pump. The rate of heat loss from the wire is equal to the power P supplied to it, and is given in watts by the product of the current and the voltage. A curve is plotted with pressure as abscissa and P as ordinate. The curve shows (1) that heat transfer through a gas by convection decreases as the pressure decreases and vanishes below a certain pressure; (2) that conduction by a gas is constant over a certain range of pressures but decreases when the pressure is low. At the lowest pressure reached by a Hyvac pump the heat loss is mainly by radiation and conduction through the leads. Numerical values depend upon the apparatus used and, hence, have no general importance, but the interpretation of the curve is an instructive problem for the student in the laboratory.—J. D. E.

An attempt to test Wien's radiation formula by the use of a "Lightometer" and to find the value of Planck's constant. W. O. Clarke; *Sch. Sci. Rev.* **20**, 727–730 (1939). For frequencies of the order of those of visible light and for temperatures up to about 4000°K, the Planck radiation formula reduces to $E_\nu = (8\pi\nu^3 h/c^3)e^{-h\nu/kT}d\nu$, where E_ν is the energy per unit volume between the frequencies ν and $\nu + d\nu$. If log E is plotted against $1/T$, the result should be a straight line of slope $-h\nu/k$ for fixed values of ν and width $d\nu$ of the frequency band.

A photovoltaic cell and microammeter (exposure meter or footcandle meter) is illuminated by light from a 6-v headlight lamp placed at such a distance that the meter reads full scale when the lamp is run at 7 v. A narrow frequency band is selected by the use of suitable filters, and the temperature of the lamp filament is varied over a wide range (1500° to 3000°C) by varying the voltage applied to it. A plot of $\log_{10} L$ against $1/R$, where L is the microammeter reading and R is the resistance of the lamp filament, gives a good straight line. Values of R are obtained from voltmeter-ammeter readings and can be converted into values of the absolute temperature T with the aid of a bridge measurement of the cold resistance of the filament and the assumption that R is proportional to T. The slope of the line on the graph is multiplied by $\log_e 10$ and by the factor converting $1/R$ into $1/T$, whence the value of $-h\nu/k$ is obtained. Values of this ratio for several filter combinations are plotted against the mean ν for each frequency band, resulting in a straight line of slope h/k; since k is known, h can be found. The mean of two values of h obtained in this way is 6.47×10^{-27} erg sec.—J. D. E.

A mechanical model illustrating the principle of the cyclotron. F. A. B. Ward; *Proc. Phys. Soc.* **51**, 810 (1939). In this model for lecture demonstrations or museum use, two horizontal semicircular brass plates, connected by a hinged ramp and mounted on brass sleeves which slide over vertical steel columns, are forced to execute simple harmonic motions of exactly opposite phase by a symmetrical connecting system. The oscillating level plates represent the harmonic variation of the uniform electric field within the dees of a cyclotron; the sloping ramp represents the electric gradient between the dees.

A positive ion released near one of the dees when it is positive will be accelerated to the other dee, and, if the electric oscillations and the magnetic field are adjusted for resonance, the ion will describe within the dees semi-circles whose radii increase in proportion to $\sqrt{n}$, n representing the number of accelerations the ion has received. In the model a set of grooves of radii proportional to 1, $\sqrt{3}$, $\sqrt{5}$ is cut in one plate, while a set with radii proportional to $\sqrt{2}$, $\sqrt{4}$, $\sqrt{6}$ is cut in the other. Short connecting grooves in the ramp give a continuous path made up of semi-circles of progressively increasing radii. To reproduce the motion of the ion, a ball is placed at the beginning of the innermost groove of the ramp and released when the slope is steepest.

The plates and ramp form a semi-circular disk about 11 in. in diameter, the ramp being 1 in. long. The best frequency is about 35 min^{-1} with an amplitude of about 0.18 cm. The radii of the grooves in one plate are $1\frac{9}{16}$, $2\frac{3}{4}$, $3\frac{9}{16}$, $4\frac{3}{16}$ and $4\frac{3}{4}$ in., with centers $-\frac{9}{16}$, $-\frac{3}{8}$, $-\frac{5}{16}$, $-\frac{5}{16}$ and $-\frac{1}{4}$ in. from the center of the plates; corresponding values for the grooves in the other plate are $2\frac{1}{4}$, $3\frac{3}{16}$, $3\frac{7}{8}$, $4\frac{1}{2}$ and 5 in., and $\frac{1}{8}$, $\frac{1}{16}$, 0, 0 and 0 in. The ramp has pairs of grooves of radii $1\frac{9}{16}$ and $2\frac{1}{4}$, $3\frac{3}{16}$ and $3\frac{7}{8}$, and $4\frac{1}{2}$ and 5 in., centered at $-\frac{9}{16}$, $-\frac{1}{8}$, $-\frac{1}{16}$, 0, 0 and 0 in. from the center of the plates. These grooves are of V section, $\frac{3}{16}$ in. wide at the top and $\frac{3}{32}$ in. deep. The ramp hinges have their axes of rotation in the plane of the plates with the edges of the plates and ramp leveled off to prevent relative motion.

One plate is fastened to a brass disk which is mounted on a hollow brass sleeve. This sleeve slides over a hollow steel pillar screwed to a base plate. The driving rod from the eccentric passes up through the pillar to the sleeve and is clamped to it by a set screw. The other plate rests upon a brass disk with a similar hollow brass sleeve sliding over a

second steel pillar. To allow the necessary horizontal slip of the second plate during the oscillatory motion, it is not fastened to the brass disk but is made to execute the same vertical motion as the disk by a short brass ring fastened to it and engaging the under surface of the disk. The two plates are connected by a centrally pivoted steel rocking lever bored at both ends and connected to the sleeves supporting the plates by brass forks whose stems are inserted into the ends of the lever while the prongs engage, by means of two screws, holes in the sleeves. The instrument must be leveled before operation.

The dimensions of the model are limited by friction, which has no counterpart in the actual cyclotron. This frictional loss increases with increasing circumference and also with increasing velocity until finally a point is reached where it becomes equal to the energy gained in descending the ramp and no further acceleration is possible. To reduce friction, the grooves are made narrow but they must not be too narrow or the ball will escape from its path owing to centrifugal force. A groove $\frac{3}{16}$ in. wide at the top permits a maximum speed of about 47 rev/min for a $\frac{1}{2}$-in. ball in a 5-in. circle. A second limit rises because of the discontinuity of slope at the hinge; the ball bounces down the ramp and does not receive the full energy increment. Because of this effect, most troublesome when the ball is moving rapidly, an increase of amplitude produces relatively little increase in the frequency at which the model operates best.—H. N. O.

Check List of Periodical Literature

Some mathematical aspects of motion and causality. H. J. Ettlinger; *Scripta Math.* **6,** 141–148 (1940). A discussion of the interplay of certain physical and mathematical models.

A scientific search for the secret of Stradivarius. F. A. Saunders; *J. Frank. Inst.* **229,** 1–20 (1940). Gives the results of a study of violin properties and performance. Modern craftsmen produce instruments that are probably as well made as any ever were; if there are any properties that new wood lacks, their mechanical character can be discovered and methods can be devised to produce them in months instead of centuries. To a race that has learned how to fly and to communicate across the oceans, so simple a problem as that of making excellent violins seems almost absurdly easy.

Appeal for fundamental research. H. Hoover; *J. App. Phys.* **10,** 688–691 (1939). Essence of an address delivered at Northwestern University.

Recent advancements in applied spectroscopy. W. F. Meggers, R. A. Sawyer, W. R. Brode, G. R. Harrison, H. M. Randall, G. H. Dieke; *J. App. Phys.* **10,** 734–816 (1939). Six papers on apparatus, atomic and molecular emission spectra, absorption spectra, infra-red spectroscopy and astronomical spectra.

Physics in 1939. T. H. Osgood; *J. App. Phys.* **11,** 2–17 (1940). A survey.

Physics of photography. J. H. Webb, T. R. Wilkins, D. L. MacAdam, R. Kingslake; *J. App. Phys.* **11,** 18–69 (1940). Four articles on the physical basis of image formation, effect of atomic particles on photographic emulsions, color photography and wide aperture lens.

Electrical conductivity of metals. J. Bardeen; *J. App. Phys.* **11,** 88–111 (1940). A survey.

Education by authority or for authority? Are science teachers teaching science? O. F. Curtis; *Science* **90,** 93–101 (1939).

New Magnetic Alloys. L. F. Bates; *Sch. Sci. Rev.* **21,** 817–830 (1939). An elementary discussion of the composition, properties and uses of Permalloy, mu-metal, Perminvar and other magnetic alloys.

Entropy and the universe. R. E. D. Clark; *Sch. Sci. Rev.* **21,** 831–846 (1939). Discusses the second law of thermodynamics as a special case of the fundamental "law of morpholysis" and adduces arguments against the "chance" theory of the universe.

The plight of science education. Anon.; *Cenco News Chats,* No. 27, 3 (1940). It is an obligation of both the educationist and the scientist to seek the ground of common agreement concerning problems of science education.

Industry, and science education. Anon.; *Cenco News Chats,* No. 28, 3 (1940). Unless the high school student has the opportunity to study physics as physics and chemistry as chemistry, his choice of a career when he enters college will likely be away from rather than toward these sciences.

Views on machinery and unemployment. C. E. Dankert; *Sci. Mo.* **50,** 155–162 (1940). An economist presents a set of general conclusions concerning technologic unemployment based on historical and present-day evidence.

The use, care and maintenance of laboratory instruments. H. N. Hayward; *J. Eng. Ed.* **30,** 506–510 (1940). Brief remarks on electrical instruments.

Photoelastic demonstrations as teaching aids in strength of materials. M. M. Frocht; *J. Eng. Ed.* **30,** 567–582 (1940). The photographs of various stress patterns included in this article are suitable for lantern slides.

The role of chance in discovery. W. B. Cannon; *Sci. Mo.* **50,** 204–209 (1940). Noteworthy examples of serendipity in the sciences.

The space in which we live. P. R. Heyl; *Sci. Mo.* **50,** 251–257 (1940). Empty space is a greater wonder than its visible contents.

Science in an unfriendly world. W. J. Lyons; *Sci. Mo.* **50,** 258–263 (1940). It is not advocated that scientists organize to entrench modern science as an established system, simply for the sake of self-perpetuation, but that they face the real problem, which is the preservation of a proven method of inquiry against the suppression of all significant inquiry.

Development and manufacture of optical glass in America. M. H. Eisenhart and E. W. Melson; *Sci. Mo.* **50,** 323–334 (1940).

Science teaching at the Liverpool Institute, 1835–1852. C. Foster; *J. Chem. Ed.* **17,** 136–138 (1940). The Liverpool Institute was the first English school to add natural science to the curriculum.

A lantern demonstration of a rotating-vibrating diatomic molecule. C. L. Wilson; *J. Chem. Ed.* **17,** 187–189 (1940). A simple rackwork slide which represents rotation, vibration and rotation-vibration in unsymmetrical diatomic molecules is described.